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On the importance of the *upside down* test in absolute socioeconomic health inequality comparisons[*]

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Abstract

This paper shows that it is impossible to obtain a robust ranking of absolute socioeconomic health inequality if one only imposes Bleichrodt and van Doorslaer's (2006) *principle of income-related health transfer*. This means that for any comparison, some indices obeying this ethical principle will always contradict the ranking produced by other indices obeying the same ethical principle. This results points to the need to impose more ethical structure on indices when one wants to identify robust rankings of absolute socioeconomic health inequality. We show that imposing Erreygers, Clarke and Van Ourti's (2012) *upside down test* allows for the identification of robust orderings of absolute health inequality. We also show that alternatively one can increase inequality aversion and impose *higher order pro-poor principles of income-related health transfer sensitivity*. In order to make the identification of all robust orderings implementable using survey data, the paper also discusses statistical inference for these positional dominance tests. To illustrate the empirical relevance of the proposed approach, we compare joint distributions of income and a health-related behavior in the United States in 1997 and 2014.

JEL Classification: D63; I10

Keywords: Generalized health concentration curves; generalized health range curves; absolute socioeconomic health inequality; stochastic dominance; inference

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1 Introduction

Measuring socioeconomic health inequality is central for researchers who wish to monitor the evolution of the distribution of health outcomes and health-related behaviors within and between countries/sociodemographic groups (or any two distributions). The most widely-used measures of socioeconomic health inequality are rank-dependent measures that can be expressed in absolute terms as well as in relative terms. While an absolute measure of socioeconomic health inequality retains the same units as the variable of interest and provides information on the level of the change, a relative measure of socioeconomic health inequality quantifies percentage differences in this health outcome. Generally speaking, evidence from the empirical literature shows that analyses based on absolute and relative measures of health outcomes may not lead to the same conclusions. This same issue arises when measuring socioeconomic health inequality. Absolute and relative rank-dependent measures may not lead to the same rankings and thus may result in different policy recommendations. While it is not necessary that both measures point in the same direction, targeting policies may sometimes be desirable if based absolute (relative) health measures rather than relative (absolute) health measures when these two measures diverge. This is why the World Health Organization's (WHO) commission on social determinants of health recommends reporting both relative and absolute measures of health inequalities when feasible. Following up on this recommendation, King, Harper and Young (2012) assess whether these guidelines are followed in research on socioeconomic health inequality. Evidence from their study shows that 75% of empirical research use relative measure of health inequality, 18% of empirical research use absolute inequality measures and that 7% report both relative and absolute ones. Based on this evidence, it is clear that while rank-dependent measures of socioeconomic health inequality are well-established in this literature, the question on whether distributions should be compared based on an absolute or relative indices remains open to debate with evidence of a preference for reporting relative health inequality results. As for the literature in health economics, to the best of our knowledge, there are only two papers that consider both absolute socioeconomic and relative health inequality indices in their analyses: Erreygers, Clarke and Zheng (2017) and Makdissi and Yazbeck (2017). These two papers adopt an index-based approach to measure socioeconomic inequality.

There are two approaches to socioeconomic health inequality analysis: and index-based approach and a dominance based approach. An index-based approach (usually) provides a complete ordering of these inequalities but these rankings depend heavily on the mathematical form specified when computing these inequalities. A dominance-based approach to inequalities overcomes this issue as it provides robust (but partial) ordering that do not depend on the specific mathematical form for the inequality index. Most of the literature on dominance and health inequality focused on deriving conditions for robust orderings of relative socioeconomic health inequalities. Exceptions are are noted for cases where the variable of interest is not ratio-scale (i.e., for which zero is not well-defined) as in the work of Allison and Foster (2004) for pure health inequality and the work Makdissi and Yazbeck (2017) in the context of socioeconomic health inequality. Nevertheless, because of their focus on ordinal data both of these papers define a dominance-based approach in the context of an index-based approach rather than in the canonical way.[1] Thus, the literature on dominance-based approach is still silent on how to provide robust orderings of absolute socioeconomic health inequalities.

The overarching objective of this paper is to fill the gap in the literature on robust rankings of health distribution as well as participate in the debate on the importance of the reporting (or lack of reporting) of absolute measures of socioeconomic health inequality in the context of ratio scale health variables. It investigates the properties of absolute

[1] The necessity of the use of a dominance-based approach arises because of the uncertainty around the mathematical form that one should impose on the scale used to transform ordinal data rather than uncertainty around the specific mathematical form of the index itself.

socioeconomic health inequality indices (and their implications) when used to produce robust rankings of health distributions and uncovers a new impossibility result for absolute socioeconomic health inequality rankings. By studying this impossibility result, this paper sheds light on the interdependence between the ethical principles of socioeconomic health inequality indices and the feasibility of robust rankings of health distributions in the case of absolute inequality. It then proposes the necessary measurement and statistical tools that allow the researchers to overcome this impossibility result and thus provides solutions that make it possible to follow the WHO recommendation by reporting rankings of both relative and absolute health inequalities.

This paper contributes to the literature on the measurement of health inequality in three distinct ways. First, it puts forward a new impossibility result arising in absolute socioeconomic health inequality rankings. This impossibility result uncovers the fact that one cannot obtain any robust ranking of absolute health inequalities at the second order. This means that while it may be feasible to compute an absolute index of socioeconomic health inequality numerically, the rankings provided will be arbitrary. This impossibility result also sheds light on the importance of caution when applying recommendations of the WHO regarding reporting both absolute and relative measures. Thus, without further assumptions, policy recommendations based on the use of absolute inequality measure will depend heavily on the specific index selected by the researcher. As no one has discussed this issue before, we reckon that this impossibility result may be one of the reasons behind the low use of absolute health inequality indices observed in the literature. Second, given the existence of this impossibility result, the paper contributes to the literature by developing the necessary conditions for robust orderings at the second order and necessary conditions for higher orders for generalized concentration curves. It also provides guidelines for the necessary conditions required for the existence of these higher order robust orderings. Third, given that the ethical principles play a central role in addressing the impossibility result, this paper contributes to the literature and the ongoing debate on the ethical principles that indices should obey by offering a unified approach for dominance conditions in the case of absolute measures of socioeconomic health inequality. Whether the *principle of income-related health transfers* alone is appropriate or whether it should be complemented with *symmetry around the median principle* is still open to debate. This paper provides evidence in support of the crucial importance of imposing the *symmetry around the median principle* (in addition to the canonical ethical principle) when absolute socioeconomic health inequality rankings are impossible.

The remaining of this paper is organized as follows. In Section 2, we provide a brief review of the literature on the relative and absolute measures of socioeconomic health inequality. In Section 3, we establish the measurement framework in which we operate as well as the ethical principles governing these measures properties. In Section 4, we provide conditions under which robust orderings of joint distributions of health and income can be identified. In Section 5, we discuss the estimation and inference corresponding to the methods developed in Section 4. In Section 6, we provide an empirical illustration using information on cigarette consumption and overweightedness from the National Health Interview Survey (NHIS) in 1997 and 2014. Finally, in Section 7, we conclude.

2 Literature Review

This paper is related to two main strands of the literature on measurement of socioeconomic health inequality; the literature on robust orderings of health distributions and the literature on the ethical principles underlying socioeconomic health inequality indices.

Generally speaking, the literature on robust orderings of health distributions consists of identifying conditions under which the ranking of any two joint distributions of health and income does not depend on the specific mathematical form of the indices used to produce these rankings. In a seminal work on dominance and health inequalities Allison and Foster (2004) develop conditions for robust comparison of health distributions for categorical variables in the context of absolute pure health inequality indices. Their work exploits uncertainty regarding the mathematical form of the scale imposed on the ordinal health variables. While their paper provides robust orderings, it does not account for the socioeconomic dimension of health which is an important dimension that is of great interest for policy makers. To account for this possibility, Makdissi and Yazbeck (2017) derive tests to identify robust rankings of health achievement indices as well as absolute and relative socioeconomic health indices. However, it should be noted that both of these papers focus on rankings that are robust to the specified numerical scale rather than the specific form of the indices.[2] As for the literature on robust orderings of socioeconomic health inequality that exploits dominance in the canonical way (i.e. to account for uncertainty around the mathematical form of the index), it mainly focused on relative health inequality. More specifically, building on Makdissi and Yazbeck (2014), Khaled, Makdissi and Yazbeck (2018) provide a unified approach using health concentration curves and health range curves (for relative socioeconomic health inequality) as well as their generalized versions (for health achievement). The authors develop conditions for robust rankings that are valid for all health achievement or relative socioeconomic health inequality indices but do not consider the case of absolute measures of socioeconomic health inequality. From this perspective, the current paper fills this gap in the literature on robust orderings by providing the conditions under which absolute socioeconomic health inequality measures can lead to robust rankings. Developing these conditions, uncovers the presence of an impossibility result that is not acknowledged in the literature. To overcome this impossibility result, we propose

[2] Using a different approach to Makdissi and Yazbeck (2014) offer a framework for socioeconomic health inequalities using categorical health variables.

two paths that depend heavily on the ethical position of the researcher and/or the level of aversion to socioeconomic health inequality she/he is willing to impose. If the researcher ethical principles are compatible with *principles of pro-poor health transfer sensitivity* and is willing to impose a higher level of aversion to socioeconomic health inequality, then (under certain conditions) increasing the order of dominance may lead to robust ranking.[3] Otherwise, if these conditions are not met or if the researcher is not willing to increase the order of aversion to socioeconomic health inequality, she/he will need to revisit the ethical principles of the indices. Indeed, part of literature on the measurement of socioeconomic health inequalities focused on the ethical principles underlying these indices. Some assume that health researchers should be concerned with inequalities that occur in the lower part of the distribution of socioeconomic status (Wagstaff, 2002) and others suggest that the analyst may be more concerned with deviations occurring away from the median of the socioeconomic status (Erreygers, Clarke and Van Ourti, 2012). While the desirable ethical principles for these measures are still open to discussion, this paper contributes to this debate by providing evidence on the central role of the *symmetry around the median principle* in identifying robust rankings based on absolute socioeconomic measures of health inequality. As such, this paper sheds light on the interdependence between the ethical principles of socioeconomic health inequality indices and the feasibility of robust rankings of health distributions in the case of absolute inequality. While the adoption of different ethical principles may lead to different robust rankings, this paper shows that, in the case of absolute socioeconomic health inequality rankings, the *income-related health transfer* principle is not a sufficient (bar very specific cases). A natural solution for this issue is to exploit the properties of an established ethical principle that can provide robust ranking at the second order. Conveniently, adding the *symmetry around the median* principle (Erreygers, Clarke

[3] Later in the paper we will explain these conditions in details, but graphically these conditions are met if there is an intersection of the two concentration curves at the second order.

and Van Ourti, 2012) to the *principle of income-related health transfer* offers a practical solution for this issue.

3 Measurement framework

The purpose of this section is to develop a measurement framework for absolute socioeconomic health inequality indices. To provide the necessary background, we will first discuss the differences between absolute and relative measure of socioeconomic health inequality. We will then discuss absolute health inequality indices in details.

3.1 Absolute and relative socioeconomic health inequality indices

Relative socioeconomic health indices and absolute socioeconomic health indices are functionals of the joint distribution of health, H and income, Y Let H and Y be 2 random variables that are absolutely continuous with support on the positive half real line and with densities f_H and f_Y respectively.[4] Let $f_{Y,H}$ be the joint density of the 2 random variables and $F_Y(y)$ be the cumulative distribution of income. Let $h(p)$ be the conditional expectation of health, H, with respect to Y equal to its p-quantile. Formally,

$$h(p) = E[H|Y = F_Y^{-1}(p)]$$

We measure absolute socioeconomic health inequality in a rank dependent framework where ranks are individuals's position in the distribution of socioeconomic statuses. This absolute socioeconomic health inequality can be interpreted as the cost of socioeconomic inequalities in health.[5] Formally, these indices can be written as

$$I_A(h) = \int_0^1 \nu(p)h(p)\mathrm{d}p, \tag{1}$$

where $\nu(p)$ is a social weight function. The assumptions made on this social weight function will be discussed in the next section. If we divide the absolute socioeconomic health

[4]In this paper, we assume that this health measure is a ratio-scale variable.

[5]For more information on this interpretation see Khaled, Makdissi and Yazbeck (2018).

inequality index by the average health status $\mu_h = \int_0^1 h(p)\mathrm{d}p$ we get a relative measure of socioeconomic health inequality $I_R(h) = I_A(h)/\mu_h$ more formally this index can be written as follows:

$$I_R(h) = \int_0^1 \nu(p)\frac{h(p)}{\mu_h}\mathrm{d}p. \tag{2}$$

It is important to note that the mathematical properties imposed on the social weight function $\nu(p)$ for relative socioeconomic health inequality indices are the same as the ones imposed on the absolute socioeconomic health inequality indices however their implications as far as the properties of the indices are concerned are different.

3.2 Principle of income-related health transfer

The social weight function $\nu(p)$ in equations (1) and (2) satisfies the following assumptions:

A.1 $\nu^{(1)}(p) > 0$

A.2 $\int_0^1 \nu(p)\mathrm{d}p = 0,$

where $\nu^{(i)}(p) = \frac{\partial^i \nu(p)}{\partial p^i}$. Assumptions A.1 is embedded in Bleichrodt and van Doorslaer (2006) *principle of income-related health transfer* where the contribution of an individual health status to socioeconomic health inequality is non-decreasing with socioeconomic status. Keeping everything else constant, this means that if the relatively healthier are the rich (poor), then socioeconomic health inequality will be higher (lower). In addition, this principle implies that performing a mean preserving health transfer δ_h from an individual at a higher socioeconomic rank to an individual at a lower socioeconomic rank decreases socioeconomic health inequality.

Assumption A.2 guarantees that the weight function $\nu(p)$ sums to zero (i.e., $\int_0^1 \nu(p)\mathrm{d}p = 0$) and that absolute inequality indices have two basic desirable properties. The first desired property requires the inequality indices to be equal to zero ($I_A(h) = 0$) if everyone has the same health status, $\overline{h}$. The second property requires $I_A(h)$ remains unchanged if everyone's health increases by the same magnitude. It is important to note that for the relative indices of socioeconomic inequality Assumption A.2 garanties that I_R remains unchanged if everyone's health increases by the same proportion. This is considered to be a desirable property for a relative index of inequality.

$I_A\,(.)$ and $I_R(.)$ are considered to be rank dependant measures of socioeconomic health inequality when the social weight function satisfies assumptions A.1 and A.2. Let us denote by Λ_A^2 the set of all rank dependent absolute socioeconomic health inequality indices and obeying these two assumptions. Formally, we can define:

$$\Lambda_A^2 := \left\{ I_A(h) \left| \begin{array}{c} \nu(p) \text{ is continuous and differentiable almost} \\ \text{everywhere over } [0,1]\,, \int_0^1 \nu(p)\mathrm{d}p = 0, \\ \nu^{(1)}\,(p) > 0,\ \forall p \in [0,1] \end{array} \right. \right\}.$$

Also let us denote by Λ_R^2 the set of all rank dependent relative socioeconomic health inequality indices obeying assumptions A.1 and A.2. Formally, we can define:

$$\Lambda_R^2 := \left\{ I_R(h) \left| I_R(h) = \frac{I_A(h)}{\mu_h} \wedge\ I_A(h) \in \Lambda_A^2 \right. \right\}$$

3.3 Symmetry around the median

Erreygers, Clarke and Van Ourti (2012) suggest that in addition to Assumptions A.1 and A.2 there is a desirable property that a measure of socioeconomic health inequality should have; it should pass the *upside down* test. For $g(p) = h(1-p)$, the *upside down* test consists in verifying if $I_A\,(g)$ is always positive (negative) when $I_A\,(h)$ is negative (positive). Erreygers, Clarke and Van Ourti (2012) show that an index of socioeconomic health inequality passes this test only if its weight function $\nu(p)$ is symmetric around the median of socioeconomic statuses ($p = 0.5$). This means that the following assumption

A.3 $\nu(1-p) = -\nu(p)$

should hold. It should be noted that assumption A.3 also implies that $\nu(0.5) = 0$. Let $\Lambda_{A\rho}^2 \subset \Lambda_A^2$ be the subset of rank dependent absolute socioeconomic health inequality indices that pass the *upside down* test. It is possible to define these subsets as follows:

$$\Lambda^2_{A\rho} := \left\{ I(h) \in \Lambda^2_A \mid \nu(1-p) = -\nu(p) \ \forall p \in [0,1] \right\}.$$

4 Identifying robust orderings of health distributions

In the previous section, we have compared relative and absolute indices of socioeconomic health inequality indices. These indices are summary measures for relative and absolute inequalities and are graphically depicted by the Concentration Curve and the Generalized Concentration Curve respectively. We have also discussed the different ethical principles that the analyst may impose on these two types of indices. In this section, we provide necessary conditions to identify robust rankings of absolute socioeconomic health inequality. This means, we identify the rankings obtained by rank dependent absolute inequality indices that are not contingent to the specific mathematical form of the index. We will propose new positional dominance tests for rankings of absolute socioeconomic health inequality for subsets of indices obeying the symmetry around the median principle. As we will explain in more details below, under the *principle of income-related health transfer* alone, the standard generalized (or absolute) health concentration curves cannot be be used to identify robust rankings of absolute socioeconomic health inequalities (at the second order).

4.1 Principle of income-related health transfer

Before discussing the case for the generalized (or absolute) health concentration curve, it is useful to introduce some background on the heath concentration curve as it will be a useful in understanding the limitations of the generalized (absolute) health concentration curve and the advantages of the generalized health range curve that we are proposing in this paper. The health concentration curve $C_i(p)$ plots the cumulative proportion of total health in the population i against the cumulative proportion of individuals ranked by their socioeconomic statuses. It is a graphical representation of relative inequality and is formally defined on the interval $[0, 1]$ as follows:

$$C_i(p) = \frac{1}{\mu_h} \int_0^p h_i(u) du. \tag{3}$$

Makdissi and Yazbeck (2014) explain how concentration curves can be used to identify orderings of distributions that are robust for rank dependent relative socioeconomic health inequality indices. They show that $I_R(h_1) \leq I_R(h_2)$ for all $I_R(h) \in \Lambda_R^2$ if and only if

$$C_1(p) \geq C_2(p) \text{ for all } p \in [0, 1], \tag{4}$$

The generalized (or absolute) health concentration curve $GC_i(p)$ displays the absolute contribution of the p poorest individuals to average health. In other words, its value indicates the average health that would be attained if total health was only the sum of the health of these p poorest individuals. Formally, the generalized health concentration curve $GC_i(p)$ associated with distribution $f_{Y,H}^i$ is defined over the interval $[0, 1]$ as:

$$GC_i(p) = \int_0^p h_i(u) \mathrm{d}u \tag{5}$$

It can also be written as where $GC_i(p) = \mu_{h_i} C_i(p)$. In addition to providing a graphical representation of the distribution of health statuses, the generalized health concentration curve (just like the concentration curve) can be used to derive a condition that identifies robust rankings of absolute health inequality, i.e. rankings that will remain the same for all rank dependent absolute socioeconomic health inequality indices $I_A(h(p)) \in \Lambda_A^2$.

Theorem 1 *Let $f_{Y,H}^1$ and $f_{Y,H}^2$ represent two joint densities of income and health. $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_A^2$ if and only if*

$$GC_1(p) \geq GC_2(p) \textit{ for all } p \in [0, 1],$$

and,

$$\mu_{h_2} \geq \mu_{h_1},$$

where average health status in population i is $\mu_{h_i} = \int_0^1 h_i(p)\mathrm{d}p$. Comparing Theorem 1 to the corresponding theorem on relative socioeconomic health inequality comparisons provided in Makdissi and Yazbeck (2014) and summarized in (4), we note that identifying robust orderings of absolute socioeconomic health inequality requires an additional condition that necessitates a comparison between the average health status of the two distributions. This additional condition is more restrictive than it may first appear. It implicitly means that the identification of robust rankings of absolute socioeconomic health inequality is only possible when two distributions have exactly the same average health status.

The intuition underlying this negative result is as follows. Assume that distribution 2 is obtained from distribution 1 by increasing the conditional expectation of health status, $h_1(p)$ by δ_h (i.e., $h_2(p) = h_1(p) + \delta_h$) on an interval $[p_0, p_0 + \varepsilon]$ for $\varepsilon > 0$. In this case, $\Delta I_A(h_1, h_2) = I_A(h_2) - I_A(h_1) = \delta_h \int_{p_0}^{p_0+\varepsilon} \nu(p)\mathrm{d}p$. The sign of $\Delta I_A(h_1, h_2)$ is entirely determined by $\int_{p_0}^{p_0+\varepsilon} \nu(p)\mathrm{d}p$. This means that ranking of these two populations will entirely depend on the social weight. Given that we are only imposing the *principle of income-related health transfer*, the only structure imposed on the weight function $\nu(p)$ is the non-negative slope assumption (i.e., A.1) and that both positive and negative values of the weight function sum up to zero (i.e., A.2). One issue arising from these assumptions resides in the fact that they are silent on the value of the threshold $\widetilde{p}$ at which the social weight switches from negative to positive. This means that there will always be social weight functions obeying A.1 and A.2 such that $\int_{p_0}^{p_0+\varepsilon} \nu(p)\mathrm{d}p > 0$ and other social weight functions such that $\int_{p_0}^{p_0+\varepsilon} \nu(p)\mathrm{d}p < 0$. Given that this example can be generalized to any distribution $f^i_{Y,H}$ with $\mu_{h_i} \neq \mu_{h_1}$, it is impossible to have robust rankings of distributions in terms of absolute socioeconomic health inequality for two distributions with different average health statuses.[6]

[6] It is important to note that $f^i_{Y,H}$ can be obtained by a series of positive and negative changes similar to the one discussed in the example above.

Corollary 1 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health.* $I_A(h_1) \leq I_A(h_2)$ *for all* $I(h) \in \Lambda^2_A$ *if and only if* $\mu_{h_1} = \mu_{h_2}$ *and* $I_R(h_1) \leq I_R(h_2)$ *for all* $I_R(h) \in \Lambda^2_R$.

Corollary 1 implies that if two distributions, $f^1_{Y,H}$ and $f^2_{Y,H}$, don't have the same average health status, then there will always be some indices $I_A(h) \in \Lambda^2_A$ indicating that absolute socioeconomic health inequality in $f^1_{Y,H}$ is higher than in $f^2_{Y,H}$ and other indices $I'_A(h) \in \Lambda^2_A$ indicating that absolute socioeconomic health inequality in $f^1_{Y,H}$ is lower than in $f^2_{Y,H}$. This means that if the researcher imposes the *principle of income-related health transfer* only, any ranking of absolute socioeconomic health inequality is arbitrary.

Given the impossibility result derived above, one may wonder whether it is still possible to obtain robust orderings in the context of absolute inequality and whether one can follow the WHO guidelines by reporting rankings obtained using both types of inequalities. To answer this question we examined the reason underlying this impossibility result and highlighted that the main cause for this negative result is the absence of an anchor point (i.e., $\tilde{p}$) at which the social weight function switches from negative to positive. This means without further assumptions on the social weight function it is impossible to follow WHO recommendation by reporting both absolute and relative inequality rankings. Conveniently, the importance of anchor points was discussed in the literature on ethical principles of socioeconomic health inequality indices by Erreygers, Clarke and Van Ourti (2012) who propose the *symmetry around the median principle.* This ethical principle offers a natural solution for the impossibility result as it implicitly specifies an anchor point for the social weight function at the median of socioeconomic ranks. Building on this insight we derive, in what follows, the conditions under which one can obtain robust orderings in the context of absolute socioeconomic health inequality at the second order using Erreygers, Clarke and van Ourti's ethical principles.

4.2 Symmetry around the median principle

If the researcher is willing to impose more structure on the properties of the social weight function, then it is possible to derive the necessary conditions for a robust ordering of absolute socioeconomic health inequality comparisons. By imposing the symmetry around the median principle, the researcher implicitly fixes the value of the threshold at $\widetilde{p} = 0.5$ which offers a solution to the arbitrariness of absolute socioeconomic health inequality described earlier. In this case, if there is an increase in the conditional expectation of health status, $h_1(p)$ by δ_h on an interval $[p_0, p_0 + \varepsilon]$ and if this interval in entirely located below (above) $\widetilde{p} = 0.5$, absolute socioeconomic health inequality decreases (increases) for all indices passing this test.

As a result, once we impose the symmetry of the social weight function, it is possible to identify robust rankings of absolute socioeconomic health inequality for indices obeying the *principle of income-related health transfer* and *symmetry around the median* by exploiting Khaled, Makdissi and Yazbeck's (2018) generalized health range curves. The generalized health range curve, $GR(p)$ represents the cumulative health range at rank p. Let us define the range of health statuses at rank p as $r(p) = h(1-p) - h(p)$, the generalized health range curve $GR_i(p)$ associated with distribution $f^i_{Y,H}$ is formally defined over the interval $[0, 0.5]$ as:

$$GR_i(p) = \int_0^p r_i(u)\mathrm{d}u. \tag{6}$$

Theorem 2 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda^2_{A\rho}$ if and only if*

$$GR_2(p) \geq GR_1(p) \text{ for all } p \in [0, 0.5].$$

It is important to note that Theorem 2 implies that it is possible to identify a robust orderings of absolute socioeconomic health inequality at the second order if one is willing to restrict to indices that pass Erreygers, Clarke and Van Ourti (2012) *upside down* test (i.e., obey the symmetry around the median principle).

If the analyst is not willing to only consider the indices that pass *upside down* test or if no robust ranking is obtained by testing the condition in Theorem 2, she/he may increase aversion to socioeconomic health inequality by imposing higher order principles. These higher order principles impose different weights for the same transfer if happening at different places in the distribution of socioeconomic status. As pointed in Khaled, Makdissi and Yazbeck (2018) there are two distinct views regarding what constitutes a desirable higher order principle of aversion to socioeconomic health inequality: *pro-poor health transfer sensitivity principles* and *pro-extreme ranks health transfer sensitivity principles*.

4.3 Pro-poor transfer sensitivity principles

The higher order transfer principles associated with Wagstaff (2002) *pro-poor health transfer sensitivity* approach are discussed in details in Makdissi and Yazbeck (2014). These transfer principles assume that health transfers become more desirable if they are occurring in the lower part of the distribution of socioeconomic ranks. In the context of this paper, this means that an absolute socioeconomic health inequality index obeying A.1 and A.2 obeys the s-th order *pro-poor health transfer sensitivity* if $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ for all $i = 1$ to $s-1$. We define the set of indices obeying all *pro-poor health transfer sensitivity principles* of order $i = 3$ to s as $\Lambda^s_{A\pi}$. One can identify robust rankings of absolute socioeconomic health inequality with higher order generalized health concentration curves which are defined over the $[0, 1]$ interval as

$$GC_i^s(p) = \int_0^p GC_i^{s-1}(u)\mathrm{d}u, \tag{7}$$

where $GC_i^2(p) = GC_i(p)$.

Theorem 3 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health.* $I_A(h_1) \leq I_A(h_2)$ *for all* $I_A(h) \in \Lambda^s_{A\pi}$ *if and only if*

$$GC^s_1(p) \geq GC^s_2(p) \text{ for all } p \in [0,1].$$

and,

$$\mu_{h_2} \geq \mu_{h_1}$$

It is important to mention that the use of higher order generalized concentration curve can lead to robust rankings as moving to the higher order can be viewed as moving the threshold $\tilde{p}$ farther away from the top of socioeconomic ranks and closer to zero. However, moving this threshold away from the top does not always help to obtain a robust ranking.

Corollary 2 *If* $GC^2_1(p) \geq GC^2_2(p)$ *for all* $p \in [0,1]$*, and* $\mu_{h2} \neq \mu_{h1}$*, then there is no order* s *for which* $I_A(h_1) \leq I_A(h_2)$ *for all* $I_A(h) \in \Lambda^s_{A\pi}$.

Corollary 2 indicates that a robust ranking will not be obtained and any order order s if there is no intersection at the second order.

Corollary 3 *If* $GC^2_1(p) \geq GC^2_2(p)$ *for all* $p \in [0, p_c]$, $\mu_{h_2} \geq \mu_{h_1}$*, and if* $GC^2_1(p) > GC^2_2(p)$ *over at least one part of the interval* $[0, p_c]$*, then there exist an order* s *for which* $I_A(h_1) \leq I_A(h_2)$ *for all* $I_A(h) \in \Lambda^s_{A\pi}$.

Corollary 3 indicates that a robust ranking will be obtained and some finite order s if there is an intersection of the GC^2 curves.

4.4 Pro-extreme rank health transfer sensitivity principles

Erreygers, Clarke and Van Ourti (2012) in their paper, discuss a higher order of aversion to socioeconomic health inequality that is compatible with the *symmetry around the median principle.* These higher order principles are formalized in Khaled, Makdissi and Yazbeck

(2018) and coined as *pro-extreme ranks health transfer sensitivity* approach. This transfer sensitivity principle is compatible with valuing transfers occurring away from the median of socioeconomic ranks more than transfers those occurring close to the median. In the context of this paper, this means that an absolute socioeconomic health inequality index obeying A.1 A.2 and A.3 obeys the s-th order *pro-extreme ranks health transfer sensitivity* if $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ for all $i = 1$ to $s-1$. We define the set of indices obeying all *pro-extreme ranks health transfer sensitivity principles* of order $i = 3$ to s as $\Lambda^s_{A\rho}$. One can identify robust rankings of absolute socioeconomic health inequality with higher order generalized health range curves which are defined over the $[0, 0.5]$ interval as

$$GR_i^s(p) = \int_0^p GR_i^{s-1}(u)\mathrm{d}u, \tag{8}$$

where $GR_i^2(p) = GR_i(p)$.

Theorem 4 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda^s_{A\rho}$ if and only if*

$$GR_2^s(p) \geq GR_1^s(p) \text{ for all } p \in [0, 0.5].$$

5 Estimation and Inference

Based on the theorems presented in the previous sections it is important to note that dominance conditions for indices obeying the *upside down* test impose a condition of non-intersection between the two curves whereas the dominance conditions for indices obeying *pro-poor transfer sensitivity* require an additional test on the the average health statuses that requires a union/intersection inference test.

First we will focus on the dominance test for indices obeying the *upside down* test. Assume that we have two samples S_1 and S_2 of sizes n_1 and n_2 drawn from $f^1_{Y,N}$ and $f^2_{Y,H}$. We are interested in testing if $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda^s_{A\rho}$, $s \in \{2, 3, \ldots\}$. The inference test based on the dominance result of Theorems 2 and 4 consist of testing:

$$\begin{aligned} H_0 &: GR_2^s(p) - GR_1^s(p) \geq 0, \forall p \in [0,1] \\ H_1 &: GR_2^s(p) - GR_1^s(p) < 0, \text{ for some } p \end{aligned}$$

The inspection of the above test indicates that to identify robust orders we test for dominance, i.e. when we reject H_0, we have evidence against that null of dominance of distribution of $f_{Y,N}^1$ over $f_{Y,H}^2$. While one may think that testing the null of non-dominance and establishing a case for dominance would be more intuitive, such a test requires strong evidence against the null. This strong evidence may be difficult to obtain over the entire $[0,1]$ interval (Davidson and Duclos, 2013).

Khaled, Makdissi and Yazbeck (2018) have derived the non-parametric estimators $\widehat{GR}_i^s(p)$ of $GR_i^s(p)$.

$$\begin{aligned} \widehat{GR}^s(p) &= \frac{1}{N}\sum_{i=1}^{N} h_i \frac{(\hat{F}_Y(y_i) - 1 + p)^{s-2}}{(s-2)!}[\mathbb{1}(y_i > \hat{F}_Y^{-1}(1-p))] \\ &- \frac{1}{N}\sum_{i=1}^{N} h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-2)!}[\mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p))] \end{aligned} \quad (9)$$

Let $\tau = \sup_p[GR_1^s(p) - GR_2^s(p)]$, it is straightforward to construct a KS type of test statistics $\widehat{\tau}$ that is a non-parametric estimator of τ as follows:

$$\widehat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p \left(\widehat{GR}_1^s(p) - \widehat{GR}_2^s(p)\right) \quad (10)$$

The asymptotic distribution of $\widehat{\tau}$ will that of a functional of a two-dimensional Gaussian process. To perform this test, we follow a bootstrap procedure as in Schechtman, Shelef, Yitzhaki and Zitikis (2008).[7]

Let us now turn to testing for dominance for indices obeying *pro-poor transfer sensitivity.* We are interested in testing whether $I_A(h_1) \leq I_A(h_2)$ for all $I_A(h) \in \Lambda_{A\pi}^s$, $s \in \{3, 4, \ldots\}$.

[7]Details can be found in Khaled, Makdissi and Yazbeck (2018).

The inference test based on the dominance result of Theorems 1 and 3 consist of jointly testing:

$$\begin{aligned} H_0^1 &: GC_1^s(p) - GC_2^s(p) \geq 0, \forall p \in [0,1] \\ H_1^1 &: GC_1^s(p) - GC_2^s(p) < 0, \text{ for some } p \end{aligned}$$

and

$$\begin{aligned} H_0 &: \mu_{h1} \leq \mu_{h2} \\ H_1 &: \mu_{h1} > \mu_{h2} \end{aligned}$$

Khaled, Makdissi and Yazbeck (2018) derived the non-parametric estimators $\widehat{GC}_i^s$ of GC_i^s.

$$\widehat{GC}^s(p) = \frac{1}{N}\sum_{i=1}^{N} h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-2)!} \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p)). \tag{11}$$

Testing for H_0^1 relies on $\tau = \sup_p[GC_2^s(p) - GC_1^s(p)]$. It is straightforward to construct a KS type of test statistics $\widehat{\tau}$ that is a non-parametric estimator of τ as follows:

$$\widehat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p \left(\widehat{GC}_2^s(p) - \widehat{GC}_1^s(p)\right) \tag{12}$$

However, the joint test in theorems 1 and 3 has an additional condition on the mean that needs to be tested. Accounting for this additional condition can be done by adjusting the significance level of the joint test by relying on the Holm procedure (see Lehmann and Romano, 2005; p. 348).

6 Empirical illustration

To show the empirical applicability of the approaches proposed in this paper, we conduct an empirical illustration using National Health Interview Survey data from years 1997 and 2014. This illustration provides evidence that it is impossible to obtain a robust ranking for generalized concentration curves at the second order which corroborates our theoretical findings. It also shows that it is possible to obtain these rankings when imposing the *symmetry around the median principle* in addition to the *principle of income-related health transfers*.

6.1 Data

In our illustration, we focus on comparisons of absolute socioeconomic health inequalities using two ill-health variables that have been of great interest in the health economics literature: cigarettes consumption (i.e., the number of cigarettes/day) and overweightedness. We follow Bilger, Kruger and Finkelstein (2016) and use max[0,BMI-25] as a measure of overweightedness. Given that the empirical application is mainly for illustration purposes, we will avoid drawing policy recommendations but will provide some guidance to possible interesting avenues to explore.

The NHIS monitors health outcomes of Americans since 1957. It is a cross-sectional household interview survey representative of American households and non-institutionalized individuals collected via personal household interviews. For comparison purposes, we focus on adult population in the 1997 and 2014 public use data for who we have information on income. As a result, sample sizes are 34,776 for overweightedness and 35,667 for cigarette consumption in 1997 and is 35,197 for overweightedness and 36,363 for cigarette consumption in 2014. We use the sample adult file to extract information on health-related behavior and use family income adjusted for family size to infer the socioeconomic rank of individuals.[8] In the set of inequality comparisons presented in this empirical illustration we focus on comparisons (over time) at the national level and complement it with regional comparisons for 2014.

[8] We compute equivalent income by dividing family income by the square root of household size.

6.2 Comparison of health outcomes and health related behaviors over time

Upon a quick inspection of the generalized concentration curves for cigarettes consumption in 1997 and in 2014 (Figure 1) we notice that the generalized concentration curve in 1997 (GC^2_{1997}) is above the generalized concentration curve in 2014 (GC^2_{2014}) without any intersection on the interval $[0, 1]$. This is also reflected in Table 1 where we find strong evidence against the null $H_0 : GC^2_{1997} \leq GC^2_{2014}$ (at 1% significance level) and no strong evidence against the null $H_0 : GC^2_{1997} \geq GC^2_{2014}$. However, Theorem 1 shows that dominance can only be established if conditions on the generalized curve and on the mean are met simultaneously. Looking at test results on the mean in Table 1, we notice that the there strong evidence against the null hypothesis $H_0 : \mu_{2014} \geq \mu_{1997}$. Thus, combining information from Figure 1 and Table 1, we note that it is not possible to establish any dominance result (for cigarette consumption) at the second order if we are only imposing *income-related health transfer principle*. This result is very much in line with the prediction of Theorem 1 and Corollary 1, as the condition on the mean and the condition on the generalized concentration curve cannot be met simultaneously at the second order. More specifically, mean reversal requires that the generalized concentration curves intersect, which is a violation of dominance by definition. If the researcher is willing to increase aversion to socioeconomic health inequality by increasing the order of dominance to $s = 3$ (i.e., *pro-poor transfer sensitivity principle*), it may be possible to establish dominance results if certain conditions are satisfied at the second order. In this empirical application conclusions remain unchanged as we increase the order of dominance. Indeed, as shown in Corollary 2, if there is strong evidence against the null $H_0 : GC^2_{1997} \leq GC^2_{2014}$ and no strong evidence against the null $H_0 : GC^2_{1997} \geq GC^2_{2014}$ for all $p \in [0, 1]$, then there is no order $s > 2$ for which a dominance result can be established.

Alternatively, if the researcher is willing to impose the *symmetry around the median principle*, then a dominance results can be established as there is no strong evidence against null hypothesis $H_0 : GR^2_{2014} \leq GR^2_{1997}$ and strong evidence against the null $H_0 : GR^2_{1997} \leq GR^2_{2014}$ for all $p \in [0, 1]$ for cigarette consumption (at 1% significance level) that is $I_{2014} \leq I_{1997} \forall I \in \Lambda^2_{A\rho}$. Given that we are dealing with an unhealthy behavior, a higher health range curve (or a lower inequality) in 2014 can be interpreted as more concentration of this unhealthy behavior among low socioeconomic groups. This is why we interpret this result as an indication that there is more absolute socioeconomic inequality in cigarette consumption in 2014 than in 1997.[9] Thus, in this context, an increase in absolute socioeconomic health inequality can be interpreted as an increase in the absolute value of the range between these two years reflecting either an increase in cigarette consumption of those who are below the median of socioeconomic status or a decrease in cigarette consumption for those who are above the median of socioeconomic status or a combination of both. It can also be interpreted as a decrease in cigarette consumptions for both but with a lower decrease for the poor. While this paper focuses on absolute health inequalities, it is important to highlight that these results are in the same direction as the results obtained for relative health inequalities in Khaled, Makdissi and Yazbeck (2018). So in the case of cigarette consumption both relative and absolute inequality measures are moving in the same direction.

Turning our attention to overweightedness and looking a Figure 2, we notice that the generalized concentration curves in 1997 is lower than the generalized concentration curve in 2014 without any intersection on the interval $[0, 1]$. Given that we cannot derive any conclusion based on the generalized concentration curve without checking the conditions on the means of these two distributions is met (refer to Theorem 1), we proceed by testing

[9]It should be highlighted that if we were dealing with healthy behaviors a concentration of this behavior among the lower socioeconomic groups would mean a lower socioeconomic inequality for that behavior. Indeed the interpretation of the direction of inequality depends on the nature of the health related behavior

the null hypothesis that $\mu_{2014} \leq \mu_{1997}$. Results for dominance tests presented in Table 1 show that there is not strong evidence against the null hypothesis that the average overweightedness is smaller in 2014 than in 1997. Combining information from the test on the generalized concentration curve and the test on the means indicates that it is impossible to establish any dominance at the second order when operating under the assumption of *principle of income-related health transfer*. As mentioned earlier, this result is very much in line with the prediction of Theorem 1 and Corollary 1. Increasing the order of dominance to the order 3 (i.e., *pro-poor transfer sensitivity principle*) does not change the dominance results for the generalized concentration curve as there is no intersection at the second order. This is in line with the predictions of the Corollary 2.

If the researcher is willing to operate under the assumption of *symmetry around the median principle*, then there is a strong evidence against the null hypothesis $H_0 : GR^2_{1997} \leq GR^2_{2014}$ (at 1% significance level) and we cannot reject $H_0 : GR^2_{2014} \leq GR^2_{1997}\ \forall p \in [0, 1]$. This means that there is more absolute socioeconomic health inequality in overweightedness in 2014 than in 1997 (i.e., $I_{2014} \leq I_{1997}\ \forall I \in \Lambda^2_{A\rho}$). At this point it may be informative to see whether these results are comparable to previous results obtained when applying relative measure of socioeconomic inequality. Comparing these results to the findings in Khaled, Makdissi and Yazbeck (2018), we notice that conclusions are reversed. Thus, a policy maker who looks at a relative index of socioeconomic health inequality will be lead to believe that socioeconomic overweightedness inequality has decreased between 1997 and 2014 whereas the conclusions derived based on absolute indices of socioeconomic health inequality leads to an opposite conclusion. More specifically, the relative range of overweightedness is narrower when we compare those who are below and above the median of socioeconomic status in 2014. While this conclusion is true in the case of relative socioeconomic overweightedness inequality, it is not true in the case of absolute socioeconomic overweightedness inequality.

The range of overweightedness has widened in absolute terms, which reflects an increase in the absolute differences in overweightedness between individuals that are below the median of socioeconomic status and those who are above it. This could be due to an increase in the overweightedness of those who are below the median of socioeconomic rank or a decrease of the overweightedness of those who are above the median socioeconomic rank or a combination of both. It can also be due to an increase in overweightedness for both with a greater increase for those who are below the median. More generally an increase in the absolute range of overweighteness can be interpreted as an increase in absolute socioeconomic inequality in overweightedness. This divergence in the conclusions derived from these two different measures of socioeconomic inequality is a clear example of situations where both do not point to the same direction. It highlights the importance of the availability of methods that provides robust rankings for these two types of measures as results from relative measures cannot be extrapolated to say something about absolute measures.

6.3 Regional comparison of health related behaviors

As we showed in Theorem 1, when operating under *income-related health transfer principles* the dominance condition is composite; one condition is on the curves and the other is on the mean. While the dominance condition on the generalized curves requires a non-intersection of the two generalized concentration curves, the condition on the mean can only be satisfied if the generalized curves were indeed intersecting. Given that these two conflicting conditions cannot be satisfied simultaneously we have seen that it is impossible to establish any dominance result at the second order while using generalized concentration curves. Results shown in Table 2 reflect this impossibility as there are no single dominance result reported for the class of indices belonging to Λ_A^2. However, as mentioned earlier it is possible to establish dominance results if the researcher is willing to impose a higher level of aversion to socioeconomic health inequality. While this kind of situation may theoretically occur, our data does not provide any clear empirical evidence of such an occurrence at higher orders. This is reflected in the absence of dominance results for $\Lambda^3_{A\pi}$ and $\Lambda^4_{A\pi}$ in Table 2 as well as the non dominance results in the previous section. Given the absence of empirical evidence showing that a higher level of aversion to socioeconomic health inequality may lead to dominance results, we investigated the conditions under which this is possible. We showed in Corollary 2 that as long as the intersection of the generalized curves at the second order occurs at a top income ranks, an increase in the level of aversion to socioeconomic health inequality may lead to dominance results.

As explained earlier, imposing more structure on the index may allow the researcher to establish dominance as discussed in Theorem 2 and Theorem 4. Thus, if the researcher is willing to impose the *symmetry around the median principle*, then it is possible to rank distributions and this ranking will be robust to any rank-dependent absolute socioeconomic health inequality index that passes the *upside down test*. Dominance test results reported in Table 2 show that the West has a lower absolute socioeconomic inequality in cigarette consumption than all other regions in the US at the second order and this result is statistically significant at the 1% level. This means that the range of cigarette consumption is narrower in the West reflecting either a lower consumption of those who are below the median socioeconomic status or a higher cigarette consumption for those who are above the median of socioeconomic status or a combination of both. It can also be the case that in the West, cigarette consumption increased at a slower rate for those who are below the median of socioeconomic rank than for those who are above the median of socioeconomic rank. In addition, the West has a lower absolute socioeconomic inequality in overweightedness compared to the Northeast at the second order. No further comparisons can be made with other regions unless the researcher is willing to increase the aversion to socioeconomic health inequality. If this is the case, then the West dominates the South at the third order at 5% significance level and at the fourth order at the 1% significance level. This switch between non-dominance between West and South (at the second order) to dominance of the West over the South at the third order is illustrated in Figure 3 where the confidence intervals between the two second order range curves were indistinguishable. As the level of aversion to socioeconomic health inequality increases, the two confidence intervals start to be distinct at least on the interval $[0, 0.25]$ as more weight is given to those who are further from the median of the socioeconomic rank.

7 Conclusion

Previous literature on measures of socioeconomic health inequality indices and robust comparisons of socioeconomic health inequalities focus mainly on relative measures (except for Erreygers, Clarke and Van Ourti (2017)). In this paper, we focus absolute socioeconomic health inequality comparisons in attempt to account for WHO recommendation regarding the importance of reporting both relative and absolute measures of socioeconomic health inequalities. We find that while it is possible to obtain robust rankings of relative socioeconomic inequalities, obtaining orderings of absolute socioeconomic health inequalities is more demanding. We show that there is an impossibility result arising from the conflicting conditions required when testing for dominance. We then analyze the reason behind this impossibility result and exploit an established result the literature on ethical principles; the *symmetry around the median principle* that provides a natural solution for this impossibility. This allows us to exploit analytical tools from Khaled, Makdissi and Yazbeck (2018) and establish dominance conditions for absolute socioeconomic health inequalities. In addition, the analysis of this impossibility result allowed us to establish dominance conditions using the canonical ethical principles under very specific conditions. We also provide an empirical illustration that illustrates our theoretical results which speaks to the applicability of the methods proposed in empirical studies.

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Table 1: Dominance tests for GC^s and GR^s comparisons for cigarette consumption and overweightedness

	p-value			
	cigarette cons.		overweightedness	
	s=2	s=3	s=2	s=3
$H_0 : GC^s_{1997}(p) \leqslant GC^s_{2014}(p), \forall p$ $H_1 : GC^s_{1997}(p) > GC^s_{2014}(p)$ for some p	0.0000	0.0000	0.9589	0.7357
$H_0 : GC^s_{2014}(p) \leqslant GC^s_{1997}(p), \forall p$ $H_1 : GC^s_{2014}(p) > GC^s_{1997}(p)$ for some p	0.9710	0.8448	0.0000	0.0000
$H_0 : \mu_{2014}(p) \leqslant \mu_{1997}(p)$; $H_1 : \mu_{2014}(p) > \mu_{1997}(p)$	1.0000		0.0000	
$H_0 : \mu_{2014}(p) \geqslant \mu_{1997}(p)$, $H_1 : \mu_{2014}(p) < \mu_{1997}(p)$	0.0000		1.0000	
$H_0 : \mu_{2014}(p) = \mu_{1997}(p)$; $H_1 : \mu_{2014}(p) \neq \mu_{1997}(p)$	0.0000		0.0000	
$H_0 : GR^s_{1997}(p) \leqslant GR^s_{2014}(p), \forall p$ $H_1 : GR^s_{1997}(p) > GR^s_{2014}(p)$ for some p	0.0040	0.0010	0.0080	0.0320
$H_0 : GR^s_{2014}(p) \leqslant GR^s_{1997}(p), \forall p$ $H_1 : GR^s_{2014}(p) > GR^s_{1997}(p)$ for some p	0.9620	0.8368	0.6456	0.6046

Figure 1: Socioeconomic inequality in cigarettes consumption

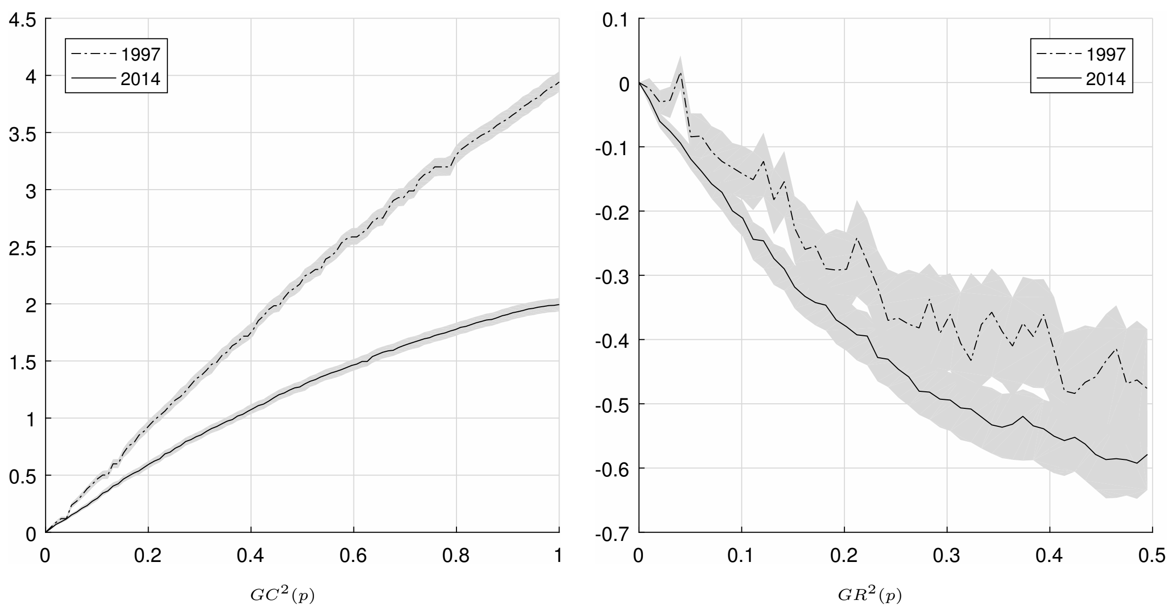

Figure 2: Socioeconomic inequality in Obesity/Overweight

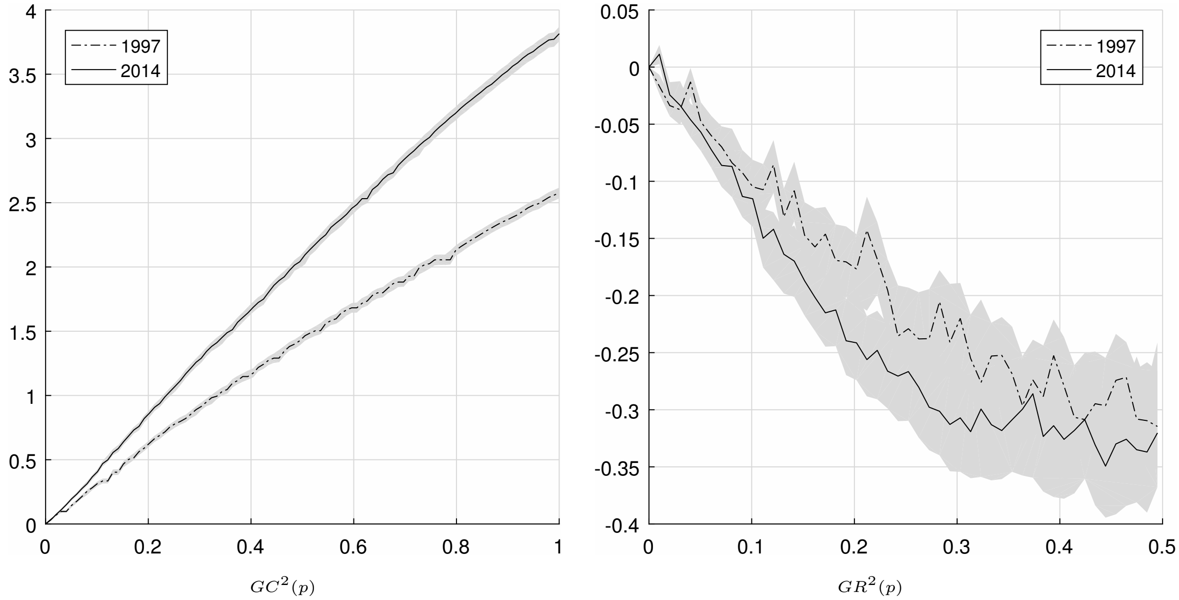

Figure 3: Socioeconomic inequality in BMI, West VS South

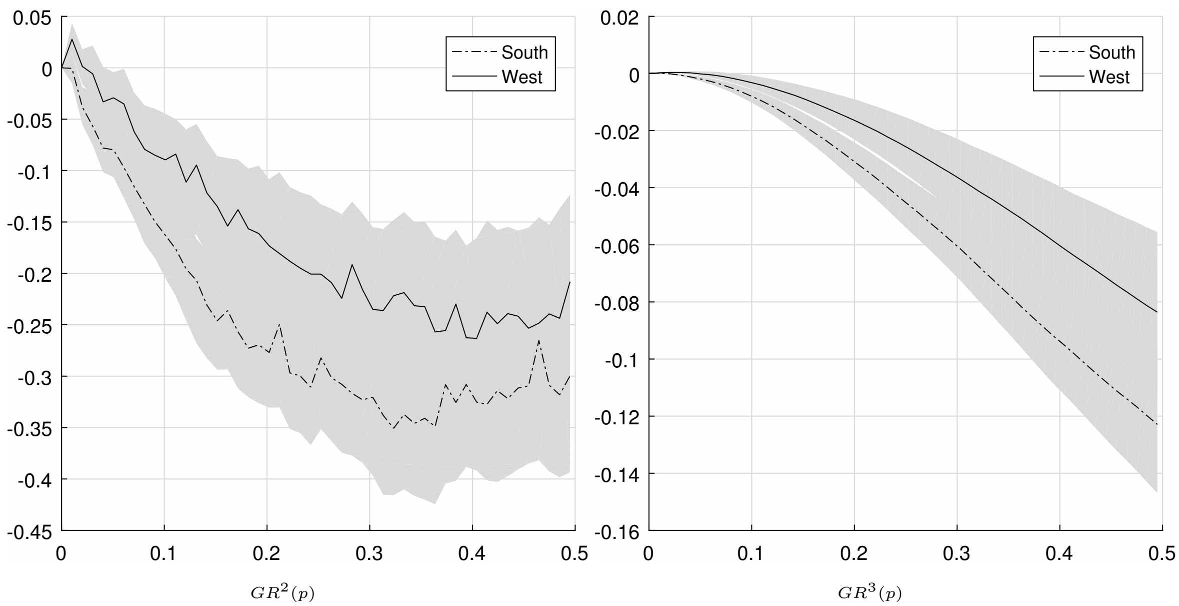

Table 2: Regional dominance tests: cigarette consumption and overweightedness

Cigarette consumption				
	Northeast	West	Midwest	South
Northeast	-		ND	ND
West	$\Lambda^2_{A\rho}$ ***	-	$\Lambda^2_{A\rho}$ ***	$\Lambda^2_{A\rho}$ ***
Midwest	ND		-	ND
South	ND		ND	-
overweightedness				
	Northeast	West	Midwest	South
Northeast	-		ND	ND
West	$\Lambda^2_{A\rho}$ ***	-	ND	$\Lambda^3_{A\rho}$ ** and $\Lambda^4_{A\rho}$ ***
Midwest	ND	ND	-	ND
South	ND		ND	-

Significance levels ** 5%; *** 1%

A Proofs of dominance theorems

For expositional ease, since Theorems 1 and 2 are particular cases of Theorems 3 and 4, we merge the proofs of Theorems 1 and 3 and of Theorems 2 and 4.

Proof of Theorem 1 and 3. Integrating by parts equation (1) yields

$$I_A(h) = \nu(p)GC(p)|_0^1 - \int_0^1 \nu^{(1)}(p)GC(p)\mathrm{d}p \tag{13}$$

Since by definition $GC(0) = 0$ and $GC(1) = \mu_h$ for all indices $I_A(h) \in \Lambda_A^2$, equation (13) can be rewritten as

$$I_A(h) = \nu(1)\mu_h - \int_0^1 \nu^{(1)}(p)GC(p)\mathrm{d}p \tag{14}$$

Now assume that for some $s-1$, equation (1) can be rewritten as:

$$I_A(h) = \nu(1)\mu_h + (-1)^{s-2} \int_0^1 \nu^{(s-2)}(p)GC^{s-1}(p)\mathrm{d}p \tag{15}$$

Integrating by parts equation (31) yields

$$I_A(h) = \nu(1)\mu_h + (-1)^{s-2} \left\{ \nu^{(s-2)}(p)GC^{s-1}(p)\Big|_0^1 - \int_0^1 \nu^{(s-1)}(p)GC^{s}(p)\mathrm{d}p \right\} \tag{16}$$

Since by definition $GC^s(0) = 0$ and $\nu^{(s-2)}(1) = 0$ for all indices $I_A(h) \in \Lambda_{A\pi}^s$, the first term in the braces on the right hand side of the equation is nil. This yield

$$I_A(h) = \nu(1)\mu_h + (-1)^{s-1} \int_0^1 \nu^{(s-1)}(p)GC^{s}(p)\mathrm{d}p \tag{17}$$

Given that equations (14) and (33) both conform to the relation depicted in equation (31), it follows that equation (33) holds for all $s \in \{2, 3, \dots\}$. From equation (33), we get

$$\Delta I_{A12} = \nu(1)(\mu_{h2} - \mu_{h1}) + (-1)^s \int_0^1 \nu^{(s-1)}(p) \left[GC_1^s(p) - GC_2^s(p)\right] \mathrm{d}p \tag{18}$$

Note that $(-1)^s \nu^{(s-1)}(p)$ is non negative. This implies that if $GC_1^s(p) \geq GC_2^s(p)$ for all $p \in [0, 1]$, then $(-1)^s \int_0^1 \nu^{(s-1)}(p) \left[GC_1^s(p) - GC_2^s(p)\right] \mathrm{d}p \geq 0$. If in addition, $\mu_{h2} \geq \mu_{h1}$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity, we need to consider three cases:

1. $\mu_{h1} > \mu_{h2}$ together with $GC_1^s(p) \geq GC_2^s(p)$ for all $p \in [0,1]$

2. $GC_1^s(p) < GC_2^s(p)$ on some arbitrary small interval $[p_c, p_c+\varepsilon]$ together with $\mu_{h1} = \mu_{h2}$

3. $GC_1^s(p) < GC_2^s(p)$ on some arbitrary small interval $[p_c, p_c+\varepsilon]$ together with $\mu_{h1} > \mu_{h2}$

Case 1: Consider the set of indices $I_A(h) \in \Lambda_{A\pi}^s$ for which $\nu^{(s-1)}(p) = (-1)^s \frac{\kappa}{GC_1(p) - GC_2(p)} \geq 0$, where $0 \leq \kappa < \nu(1)(\mu_{h1} - \mu_{h2})$. This weight function $\nu(p)$ satisfies the conditions in the definition of $\Lambda_{A\pi}^s$. From equation (34) this implies that $\Delta I_{A12} = \nu(1)(\mu_{h2} - \mu_{h1}) + \kappa < 0$. Hence it cannot be that $\mu_{h1} > \mu_{h2}$.

Case 2: First consider the set of indices $I_A(h) \in \Lambda_A^2$ for which $\nu(p)$ takes the following form:

$$\nu(p) = \begin{cases} -\kappa & 0 \leq p_c \\ \frac{\kappa+\tau}{\varepsilon}(p - p_c) - \kappa & p_c \leq p \leq p_c + \varepsilon \\ \tau & p \geq p_c + \varepsilon \end{cases} \tag{19}$$

where $p_c \in [0,1]$ and κ and τ are such that $\tau(1 - p_c - \varepsilon) - \kappa p_c - \frac{\kappa+\tau}{2}\varepsilon + 2\frac{\tau^2\varepsilon}{\kappa+\tau} = 0$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Λ_A^2. Differentiating equation (19) yields

$$\nu^{(1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ \frac{\kappa+\tau}{\varepsilon} & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{20}$$

Imagine now that $GC_1^2(p) < GC_2^2(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (19), the expression in equation (34) is negative. Hence it cannot be that $GC_1^2(p) < GC_2^2(p)$ for $p \in [p_c, p_c+\varepsilon]$ if $\mu_{h1} = \mu_{h2}$. Now consider higher order indices $I_A(h) \in \Lambda_{A\pi}^s$, $s \in \{3, 4, \dots\}$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\varepsilon & 0 \leq p_c \\ (-1)^{s-1}[p_c + \varepsilon - p] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{21}$$

where $p_c \in [0,1]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of $\Lambda^s_{A\rho}$. Differentiating equation (21) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^s & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{22}$$

Imagine now that $GC^s_1(p) < GC^s_2(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (21), the expression in equation (34) is negative. Hence it cannot be that $GC^s_1(p) < GC^s_2(p)$ for $p \in [p_c, p_c + \varepsilon]$ if $\mu_{h1} = \mu_{h2}$.

Case 3: First consider the set of indices $I_A(h) \in \Lambda^2_A$ for which $\nu(p)$ takes the following form:

$$\nu(p) = \begin{cases} -\kappa & 0 \leq p_c \\ \frac{\kappa+\tau}{\varepsilon}(p - p_c) - \kappa & p_c \leq p \leq p_c + \varepsilon \\ \tau & p \geq p_c + \varepsilon \end{cases} \tag{23}$$

where $p_c \in [0,1]$, and κ and τ are such that $\tau(1 - p_c - \varepsilon) - \kappa p_c - \frac{\kappa+\tau}{2}\varepsilon + 2\frac{\tau^2\varepsilon}{\kappa+\tau} = 0$ and $\kappa + \tau > \frac{\nu(1)(\mu_{h2}-\mu_{h1})}{\sup\{GC_2(p)-GC_1(p)\}}$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Λ^2_A. Differentiating equation (23) yields

$$\nu^{(1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ \frac{\kappa+\tau}{\varepsilon} & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{24}$$

Imagine now that $GC^2_1(p) < GC^2_2(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (23), the expression in equation (34) is negative. Hence it cannot be that $GC^2_1(p) < GC^2_2(p)$ for $p \in [p_c, p_c + \varepsilon]$ even if $\mu_{h1} < \mu_{h2}$. Now consider the set of indices $I_A(h) \in \Lambda^s_{A\pi}$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\kappa & 0 \leq p_c \\ (-1)^{s-1}\kappa\,[p_c + \varepsilon - p] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{25}$$

where $\kappa > \left(\frac{\nu(1)(\mu_{h1}-\mu_{h2})}{\varepsilon}\right)$ and $p_c \in [0,1]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of $\Lambda^s_{A\pi}$. Differentiating equation (25) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^s\kappa & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{26}$$

Imagine now that $GC_1^s(p) < GC_2^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (25), the expression in equation (34) is negative. Hence it cannot be that $GC_1^s(p) < GC_2^s(p)$ for $p \in [p_c, p_c + \varepsilon]$ if $\mu_{h1} > \mu_{h2}$. Cases 1 to 3 prove the necessity of the condition. ■

Proof of Corollary 1. We have shown that (1) $GC_1(p) \geq GC_2(p)$ for all $p \in [0,1]$ and (2) $\mu_{h2} \geq \mu_{h1}$ are necessary and sufficient condition for $\Delta I_{A12} \geq 0$ for all $I_A(h) \in \Lambda_A^2$. However, $GC_1^2(1) \geq GC_2^2(1)$ implies that $\mu_{h2} - \mu_{h1} \leq 0$ since $GC^2(1) = \mu_h$. This implies that conditions (1) and (2) are simultaneously obeyed if and only if $\mu_{h1} = \mu_{h2}$. In addition, since $GC(p) = \mu_h C(p)$, $GC_1(p) \leq GC_2(p)$ if and only if $C_1(p) \leq C_2(p)$. From Theorem 1, $C_1(p) \leq C_2(p)$ for all $p \in [0,1]$ is a necessary and sufficient condition for $\Delta I_{R12} \geq 0$ for all $I_R(h) \in \Lambda_R^2$. ■

Proof of Corollary 2. Since $GC(p)^s = \int_0^p GC^{s-1}(u)\mathrm{d}u$, $GC_1^2(p) \geq GC_2^2(p)$ for all $p \in [0,1]$ implies $GC_1^s(p) \geq GC_2^s(p)$ for all $p \in [0,1]$ for all $s \in \{3,4,\dots\}$. Since $GC_1^2(1) \geq GC_2^2(1)$ also implies that $\mu_{h2} - \mu_{h1} \leq 0$, then a ranking at any order s of *pro-poor transfer sensitivity* cannot be obtained. ■

Proof of Corollary 3. Since $GC_1^2(p) \geq GC_2^2(p)$ for all $p \in [0, p_c]$, we know that $GC_1^s(p) \geq GC_s^2(p)$ for all $p \in [0, p_c]$ for any $s \in \{3,4,\dots\}$. We also have that $GC_1^3(p_c) - GC_2^3(p_c) = a > 0$. Since $GC^s(p) = \frac{1}{(s-3)!}\int_0^p (p-u)^{s-3} GC^2(u)\mathrm{d}u$, for order s, we can write for any $p \in [p_c, 1]$,

$$\begin{aligned} GC_1^s(p) - GC_2^s(p) &= \frac{1}{(s-3)!}\int_0^p (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \\ &= \frac{1}{(s-3)!}\int_0^{p_c} (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \\ &\quad + \frac{1}{(s-3)!}\int_{p_c}^p (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \qquad (27) \end{aligned}$$

From equation (27), we know that if

$$\int_0^{p_c} (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \geq -\int_{p_c}^{p} (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u, \quad (28)$$

then $GC_1^s(p) \geq GC_2^s(p)$. We also know that

$$\int_0^{p_c} (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \geq (p-p_c)^{s-3}a. \quad (29)$$

Since the $GC_1^2(1) = \mu_{h1} \leq \mu_{h2} GC_2^2(1)$, we can infer that $-\mu_{h2} < GC_1^2(p) - GC_2^2(p)$ for all $p \in [p_c, 1]$ and

$$\int_{p_c}^{p} (p-u)^{s-3}\left[GC_1^2(u) - GC_2^2(u)\right]\mathrm{d}u \geq -\mu_{h_2}\frac{(p-p_c)^{s-2}}{s-2}. \quad (30)$$

If we choose $s \geq 2 + \frac{\mu_{h2}(1-p_c)}{a}$, we have $GC_1^s(p) \geq GC_2^s(p)$ for all $p \in [0,1]$. ■

Proof of Theorems 2 and 4. First note that for $I_A(h) \in \Lambda^s_{A\rho}$, equation (1) can be rewritten as

$$I_A(h) = -\int_0^{0.5} \nu(p) r(p)\mathrm{d}p \quad (31)$$

Integrating by parts equation (31), we get

$$I_A(h) = -\nu(p)GR^2(p)\Big|_0^{0.5} + \int_0^{0.5} \nu^{(1)}(p)GR^2(p)\mathrm{d}p. \quad (32)$$

Since by definition $GR^2(0) = 0$ and $\nu(0.5) = 0$ for all indices $I_A(h) \in \Lambda^s_{R\rho}$, the first term on the right hand side of the equation is nil. This yields to

$$I_A(h) = \int_0^{0.5} \nu^{(1)}(p)GR^2(p)\mathrm{d}p. \quad (33)$$

Now assume that for $s-1$, we have

$$I_A(h) = (-1)^{s-1}\int_0^{0.5} \nu^{(s-2)}(p)GR^{s-1}(p)\mathrm{d}p. \quad (34)$$

Integrating by parts equation (34) yields

$$I_A(h) = (-1)^{s-1}\left\{\nu^{(s-2)}(p)GR^s(p)\Big|_0^{0.5} - \int_0^{0.5} \nu^{(s-1)}GR^s(p)\mathrm{d}p\right\}. \quad (35)$$

Since by definition $GR^s(0) = 0$ and $\nu^{(s-2)}(0.5) = 0$ for all indices $I_A(h) \in \Lambda^s_{R\rho}$, the first term in the braces on the right hand side of the equation is nil. This yield

$$I_A(h) = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) R^s(p) \mathrm{d}p. \tag{36}$$

Given that equations (33) and (36) both conform to the relation depicted in equation (34), it follows that equation (36) holds for all $s \in \{2, 3, \dots\}$. Let $\Delta I_{A12} = I_A(h_2) - I_A(h_1)$. From equation (36), we get

$$\Delta I_{A12} = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) \left[GR_2^s(p) - GR_1^s(p)\right] \mathrm{d}p. \tag{37}$$

Note that $(-1)^s \nu^{(s-1)}(p)$ is non negative. This implies that if $GR_2^s(p) \geq GR_1^s(p)$ for all $p \in [0, 0.5]$, then $\Delta I_{A12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. Consider now the set of indices $I_A(h) \in \Lambda^s_{A\rho}$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\varepsilon & 0 \leq p_c \\ (-1)^{s-1}\left[p_c + \varepsilon - p\right] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{38}$$

where $p_c \in [0, 0.5]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of $\Lambda^s_{R\rho}$. Differentiating equation (38) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^s & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{39}$$

Imagine now that $GR_2^s(p) < GR_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (38), the expression in equation (37) is negative. Hence it cannot be that $GR_2^s(p) < GR_1^s(p)$ for $p \in [p_c, p_c + \varepsilon]$. This proves the necessity of the condition. ■