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Why Should Rational Smokers Find it Hard to Quit? Introducing Uncertainty into the Rational Addiction Model

Audrey Laporte[a,b], Brian Ferguson[b,c]

[a] *Institute of Health Policy, Management and Evaluation, University of Toronto*
[b] *Canadian Centre for Health Economics, Toronto, Canada*
[c] *Department of Economics and Finance, University of Guelph*

Abstract

One problem with the Becker-Murphy model of Rational Addiction, at least in the eyes of many public health specialists, is that it does not explain why so many rational, forward looking, smokers should apparently find it so hard to quit, especially since the terminal conditions are part of an intertemporal optimization problem. In this paper we apply techniques of stochastic control theory to introduce uncertainty into the individual's perception of how her stock of addiction will accumulate over time as a consequence of her time path of smoking. We assume that addiction capital is basically unobservable, so she cannot adjust her smoking behaviour according to a feedback policy rule but instead builds uncertainty into her consumption plan from the beginning. We discuss the differences between the equation explaining her lifetime smoking trajectory in the deterministic and stochastic cases, and find that the quadratic utility function which underlies the familiar lead-lag consumption form of rational addiction equation is not, in fact, capable of allowing for the type of uncertainty which we consider here.

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I. Introduction

One question which bedevils the Rational Addiction literature is why, if consumers of addictive products such as cigarettes are rational and are following an inter-temporally optimal consumption trajectory, do so many smokers find it so hard to quit? After all, the terminal conditions are part of the set of first order conditions for the optimization problem. Many rational smokers will presumably have selected their optimal trajectory with an eye to quitting at a certain date, even in the absence of a smoking-related health shock. They should, therefore, be following a cigarette consumption trajectory which is designed to reach zero consumption at some predetermined point in time in the future. Yet there seems to be a wide range of difficulty in quitting, with some smokers able to quit with no problem and others, who appear to have followed the same type of lifetime smoking trajectory as the first group, and presumably therefore have accumulated the same stock of addiction, finding it extremely difficult.

One possible explanation for the failure of the RA model to explain this phenomenon is that the standard Becker-Murphy framework is set up as a deterministic optimal control problem, whereas real life is stochastic. An individual who is making decisions today about consumption of a commodity, when today's consumption decisions have a direct influence on future behavior, is picking a consumption trajectory aimed at maximizing expected lifetime utility, not guaranteed utility. We should, therefore, set up this individual's problem as a problem in stochastic optimal control.

Furthermore, in introducing uncertainty, we need to take account of the fact that the individual's personal level of addiction is for the most part unobservable – that she might well not fully realize how addicted she is until she tries to quit. The unobservability of her stock of addiction capital adds an extra level of complication to her problem in that she cannot continuously adjust her smoking behavior in order to keep her stock of addiction on a well-defined optimal trajectory, the way she might, for example, adjust the weights on the assets in her financial portfolio if her stock of financial capital was deviating from its optimal trajectory.

That said, we have to consider how we represent uncertainty in the formal model. The standard empirical approach of specifying a trajectory of period-by-period consumption values, then adding a zero mean disturbance term at each point in time does have some appeal in the Rational Addiction context, if we assume that the individual responds to any shock which takes her off her optimal consumption trajectory by adjusting the level of consumption to put her back on that long run, rationally optimal trajectory. On the other hand, one aspect of uncertainty which seems well suited to addictive commodities, and which would be consistent with the observation that many people find it more difficult to quit smoking than they had expected, is the general notion that the variance of the uncertain term should increase the further into the future the individual is looking. By this we suggest that the magnitude of the uncertainty, in terms of the spread of possible outcomes should, when looked at from the beginning of the planning horizon, be larger the further out one is looking.

This latter approach is the one we adopt in this paper[1]. We assume that the equation of motion which determines the evolution of the individual's stock of addiction capital has, in addition to the deterministic element found in the usual theoretical presentation of the model, a stochastic element. We do this by analyzing the consumer's problem using stochastic control techniques and the Ito Calculus, and comparing key results between the deterministic and stochastic control versions of the problem. We should emphasize here that, while we will be adding a stochastic element to the equation of motion for her stock of addiction capital, we are taking this to represent the individual's uncertainty about how her own stock of addiction evolves, rather than asserting that addiction is necessarily an intrinsically stochastic variable. We will discuss this point further when we set up the stochastic version of the problem.

In the following section, we set out the version of the RA model in the form of a deterministic optimal control problem. In the third section we analyze the same problem in a stochastic control framework, using the approach originally suggested by Pindyck (1980, 1982). Pindyck's approach, while making use of a stochastic Hamiltonian and concepts from Dynamic Programming, differs from the Hamilton-Jacobi-Bellman equation approach commonly used in the literature on stochastic dynamic optimization. It has the advantage, in common with the Pontryagin approach to deterministic optimal control, of eliminating the value function and therefore obviating the need to assume a functional form for the value function. Pindyck's approach has not been widely used in the continuous time stochastic optimization economics literature, which has, on the whole, tended to take the Hamilton-Jacobi-Bellman approach, but its discrete time counterpart is well-known in the precautionary savings literature: see Dynan (1993) for example and Hori and Shimizutani (2006)[2]. Since the precautionary saving literature, like our present application, deals with choice in the face of uncertainty, the fact that the two problems can be shown to have common analytical features has a certain appeal.

Following our discussion of the Pindyck approach, we discuss the difference between the deterministic and stochastic equations of motion for the addictive consumption variable, and also discuss how our work fits into the recent literature on intertemporal choice in the presence of uncertainty. Finally, we discuss possible future directions for this line of research.

II. The Deterministic Rational Addiction Problem

As our starting point we adopt a very basic version of the RA model. The individual's instantaneous utility function is written

[1]For an alternative approach to introducing uncertainty into a somewhat related model, see Cropper (1977).

[2]Abel (1983) criticized of Pindyck's (1982) interpretation of the stochastic stationary locus and the nature of a long run – i.e. infinite horizon – equilibrium (See also Abel (1984)). These issues do not arise in our application. For related applications see Stefanou (1987), Larson (1992) and Fousekis and Shortle (1995).

(1) $U(C,S,A)$, $U_C > 0$, $U_{CC} < 0$, $U_S > 0$, $U_{SS} < 0$, $U_A < 0$, $U_{AA} < 0$, $U_{CS} = 0$, $U_{CA} = 0$, $U_{SA} > 0$

Where C is consumption of non-addictive commodities, S is consumption of an addictive commodity and A is the current level of her stock of addiction capital. C and S have the usual properties of positive and diminishing marginal utility, and for simplicity we assume separability between them. A, being a bad, yields disutility, and the amount of disutility increases as A increases. We assume separability between C and A, but we assume that $U_{SA} > 0$, so that an increase in the stock of addiction increases the marginal utility from a unit of S. This is the "strength of addiction" measure for S.

The individual has an instantaneous budget constraint, $Y = C + pS$, where Y is income and p is the relative price of S. The price of C is normalized to 1 and we assume that the budget constraint is binding at each instant of time. This last assumption allows us to write $C = Y – pS$, which can then be substituted into the utility function, giving

(2) $U(Y\text{-}pS, S, A)$

The individual's lifetime utility function is

(3) $\int_t^T U(Y - pS, S, A)\, e^{-\rho\tau} \mathrm{d}\tau$

Addiction capital evolves according to the deterministic equation of motion

(4) $\dot{A} = S – \delta A$

The individual's problem is to maximize lifetime utility with respect to choice of S, subject to (4). The current value Hamiltonian for the problem is

(5) $H = U(Y\text{-}pS, S, A) + \psi[S – \delta A]$

The first order condition for the choice of S is

(6) $U_S – pU_C + \psi = 0$

Which we re-write as

(7) $\psi = pU_C – U_S$

To find the Pontryagin necessary conditions for the dynamic problem we next differentiate (7) with respect to time, giving:

(8) $\dot{\psi} = [-p^2U_{CC} - U_{SS}]\dot{S} - U_{SA}\, \dot{A}$

Next, we write Pontryagin's equation of motion for the co-state variable, ψ:

(9) $\dot{\psi} = \rho\psi - H_A = \rho\psi - [U_A - \delta\psi]$

$= [\rho + \delta]\psi - U_A$

Which, after substituting for ψ from equation (7) and for $\dot{\psi}$ from (8) becomes

(10) $[-p^2U_{CC} - U_{SS}]\dot{S} - U_{SA}\dot{A} = [\rho + \delta][pU_C - U_S] - U_A$

Equations (10) and (4) are used to derive the phase diagram for the deterministic problem, as in Figure (1). The stationary locus for A is linear while the shape of the stationary locus for S depends on assumptions about relative magnitudes, in particular about the magnitude of U_{SA}. This last term indicates how an increase in A, the individual's stock of addiction capital, affects the marginal utility of S, the addictive commodity. It is generally taken in the RA literature as an indication of the strength of the addictive effect of S – of cigarettes, for example – since it shows how accumulated addiction capital, which has built up over time as a function both of how long the individual has been a smoker and of how much she has smoked at each instant of time, allowing for the tendency for the body to rid itself of addiction capital as reflected in the depreciation rate δ. If $U_{SA} = 0$, then S is harmful but not addictive, the case originally analyzed by Ippolito (1981).

Equation (10), in addition to permitting the derivation of the stationary locus for S, indiciates the shape of the rational addict's lifetime consumption pattern of S. We can see this if we rearrange (10), substituting for $\dot{A}$ from equation (4):

(11) $\dot{S} = \frac{[\rho+\delta][U_S - pU_C] + U_A - U_{SA}[S - \delta A]}{p^2U_{CC} + U_{SS}}$

Equation (11) will be our comparator case, showing the evolution over time of consumption of S in the non-stochastic case. Our objective here will be to assess how the introduction of uncertainty into the time path of accumulation of the individual's stock of addiction capital affects her lifetime consumption program for S.

Equation (11) is used to find the stationary locus for S in Figure 1.

FIGURE 1 ABOUT HERE

The role of a phase diagram in deterministic optimal control is to divide (S,A) space into regions in which S is rising and falling and regions in which A is rising or falling, with the stationary loci acting as boundaries between those regions. Combinations of directions of motion of S and A yield the optimal lifetime trajectory for the smoker, showing the relation between S and A at any point in time and also showing how that relation changes as the individual moves through her life course. One thing which is immediately obvious from Figure 1 is that the relation between S and A along the optimal lifetime trajectory for a finite-lived individual in the deterministic case will be non-linear: in some regions of the trajectory S will fall as A rises and in others S will rise along with A (see Ferguson (2000)). This is a consequence of the finite-horizon nature of the problem: if the individual were infinite-lived, it is much more likely that the trajectory would sketch out a monotonic relation between S and A. Assuming that our smoker is a rational, inter-temporally optimizing individual with one blind spot in that she believes she is going to live forever seems to strain the definition of forward-looking. Thus, we expect the relation between S and A to be non-linear in empirical applications in the deterministic case, and we expect that this will carry over to the stochastic case.

One characteristic of the Hamiltonian/phase-diagram approach to dynamic analysis is that it is, in essence, qualitative. In general, it does not involve solving for explicit expressions for the control variable[3]. Instead the approach gives a sense of the relation between S and A (as, for example, in our observation that it is likely to be non-linear in a finite horizon problem even if monotonic in an infinite horizon one) and a sense of what variables might shift the stationary loci and therefore change the location of the individual smoker's optimal trajectory in (S,A) space. Those insights give a general sense of what we should be looking for in an empirical smoking equation, which we could then be estimated using some kind of a flexible functional form. To derive an explicit solution expression for S as a function of A and the loci-shifting variables, would require making explicit assumptions about functional forms for the utility function and the Addiction production function[4]. Thus while the Dynamic programming and the Hamiltonian Optimal Control approaches to dynamic optimization are formally equivalent[5] one key difference between them is that in practical applications Dynamic Programming analysts tend to want to find explicit solution expressions and hence must make explicit assumptions at the very least about the functional form of the value function. Optimal Control theorists are likely to focus on the qualitative information to be derived from the phase

[3] In the RA literature it is common to assume that S can be modeled by a forward-looking second order difference equation, meaning one whose dependent variable is current cigarette consumption and whose explanatory variables include lead and lagged cigarette consumption. While this looks structural, it is in fact a dynamic reduced form equation, derived using Becker and Murphy's assumption that the individual's utility function is quadratic. Further, the empirical literature which builds on this framework seldom includes much in the way of other explanatory variables.

[4] Note that it is common to use an equation of motion for A like (4) above where the coefficient on S in the equation for $\dot{A}$ = 1 and is constant. It would make more sense, in empirical applications, to allow this to be g(S) and test for functional form. Since A is generally unobservable, we cannot, in general do so. Note also that by assuming a coefficient of 1 on S in (4) we are basically saying that Addiction Capital is measured in cigarette-equivalents. Since we have no natural unit of measurement for addiction this is probably no worse than any other assumption. Nor is it any better, in principle.

[5] In principle optimal control is slightly more general since it does not impose the same additivity requirements.

diagram, which requires assumptions about the signs, and sometimes relative magnitudes, of derivatives, but not explicit functional forms. This will become of more consequence when we discuss Pindyck's approach to stochastic dynamic optimization.

Further, considering the interpretation of the stationary loci in Figure 1, we can see that, if something happens to shift the location of one of the stationary loci, the local dynamics of the problem change. Thus if the stationary locus for S shifts, depending on the nature of the shift we could expect to see part of the region in which S was increasing, say, now become part of the region in which S is decreasing. Thus shifts in the stationary loci due to changes in some parameters of the problem reveal something about how the shape of the relation between S and A change, at least locally, as a result of the change in that parameter. While this may not yield explicit testable predictions, it may provide some guidance for empirical work.

Empirical analysis is complicated by the fact that, in the case of cigarettes, while S is observable, A, in general, is not. Some researchers have had access to data sets containing blood test information related to A, but most applied researchers do not, even when they are working with individual level data. This is the primary reason that empirical implementation of the RA framework typically takes the form of a linear equation with current consumption, S_t, as the dependent variable and lead and lag consumption values, S_{t+1} and S_{t-1} as the primary explanatory variables. The unobservability of A will become a point of some significance in what follows.

III. Stochastic Addiction Capital

As noted above, we want to consider the case in which not only is A stochastic but uncertainty about the level of A increases over time. To do this we introduce a Wiener process, or Brownian motion, to the evolution of A. The Wiener process is a continuous stochastic element and adding it essentially means that A is subject to uncertainty at every instant in time. The basic assumptions of a Wiener process are set out nicely by Mangel (1985)[6]. We define a stochastic process, Z(t), where

(i) Sample paths of Z(t) are continuous

(ii) Z(0) = 0, so we know the initial value of our variable with certainty

(iii) The increment $Z(\tau + s) - Z(\tau)$ is normally distributed with mean 0 and variance $\sigma^2 s$, where s is the length of the interval. Thus the variance of the increment in Z depends on, and increases with, the length of the period ahead, over which we are looking.

[6] See also Ferguson and Lim (1988)

In continuous time applications of Wiener processes we let s = dt, a small increment in time. The defining dZ = Z(t+dt) – Z(t), we have

(12) $EdZ = 0, E(dZ)^2 = dt, EdtdZ = 0$

In (12) the Expectations operator, E, is present because the increment dZ is a normally distributed random variable. The third of the expressions in (12) arises because dZ is on the order of magnitude of the square root of dt. Along with the assumption that $(dt)^2 = 0$, which follows from the assumption that dt is an infinitesimal, (12) constitutes the rules of multiplication of Wiener terms.

Wiener processes are continuous time processes, but because they represent continuous shocks, and therefore a process of continuous, if infinitesimal, jumps, they are not differentiable using ordinary rules of calculus. In essence, this means that dZ/dt does not exist in the usual sense. A Wiener process basically represents a variable whose time path is all corners, or spikes. Thus we have to adopt the Ito Calculus for problems involving them.

In economic applications, we are not interested in Wiener processes per se, but in the behaviour of variables which are functions of Wiener processes – i.e. variables whose behaviour over time is subject to the continuous random shocks characterized by Brownian motion. Thus define a variable x such that

(13) $dx = \alpha dt + \sigma dz$

where dz represents the Wiener process. In the absence of the dz term we would have $dx = \alpha dt$, and dividing through by dt we would have $dx/dt = \alpha$, or, in the notation used for time derivatives, $\dot{x} = \alpha$. We cannot, however, simply divide through in (13) by dt, as in

(13a) $dx/dt = \alpha + \sigma dz/dt$

Because dz/dt does not exist in the usual sense. We can, however, use the Ito calculus to analyze the effect of the Brownian motion type of uncertainty driving the Wiener process on a variable which is itself a function of x – e.g. on y = f(x,t). In doing so, note that we can take the expectation of (13):

(14) $Edx = E\alpha dt + E\sigma dz = E\alpha dt = \alpha dt$

Since σ, the variance scaling term, is non-stochastic, as are α and dt, and Edz = 0. Then we divide through by dt, giving an expression which we write as

(15) $[Edx]/dt = \alpha$.

Expression (15) refers to the expected instantaneous change in x as time passes, which we refer to as the drift term in x. Note that the drift in the stochastic x is the same as the actual time change in x if σ = 0, i.e. if x were non-stochastic. It is not, however, necessarily the actual change in x over an infinitesimal interval of time – that will be a combination of the drift plus noise.

The key to the use of the Ito calculus is that, although time derivatives may not exist in the usual sense, other derivatives do. Ito's approach to analyzing the behaviour of y is to take a second order Taylor Series expansion to df(x,t):

(16) $dy = df(x,t) = f_x(x,t)dx + ½ f_{xx}(x,t)(dx)^2 + f_t(x,t)dt + ½ f_{tt}(x,t)(dt)^2 + f_{xt}(x,t)dxdt$

Next we replace dx by $\alpha dt + \sigma dz$ everywhere, giving:

(17) $dy = f_x(x,t)[\alpha dt + \sigma dz] + ½ f_{xx}(x,t)[\alpha dt + \sigma dz]^2 + f_t(x,t)dt + ½ f_{tt}(x,t)(dt)^2 + f_{xt}(x,t)[\alpha dt + \sigma dz]dt$

Multiplying this expression out gives:

(18) $dy = f_x\, \alpha dt + f_x\, \sigma dz + ½ f_{xx}\alpha^2[dt]^2 + ½ f_{xx}\, \sigma^2[dz]^2 + ½ f_{xx}\, [2\alpha\sigma\, dt\, dz] + f_t\, dt + ½ f_{tt}\, [dt]^2 + f_{xt}\alpha[dt]^2 + f_{xt}\sigma dzdt$

Next, taking the expectations operator through and applying the rules of multiplication for Wiener processes, plus the fact that $[dt]^2$ is vanishingly small even in the absence of uncertainty, gives:

(19) $Edy = f_x\, \alpha dt + ½ f_{xx}\, \sigma^2 dt + f_t\, dt$

From which we obtain:

(20) $[Edy]/dt = [f_x\, \alpha + ½ f_{xx}\, \sigma^2 + f_t]$

Note that in the non-stochastic case we would have

(21) $dy/dt = f_x\, \alpha + f_t$

Where f_t represents any trend element in y which was not inherited from x. The drift term in (20) differs from the time trend in the non-stochastic case, as set out in (21), by the addition of the term $f_{xx}\, \sigma^2/2$. Assuming that both the non-stochastic and the stochastic examples start from the same point, the difference between the non-stochastic trend and the expected drift depends on the concavity or convexity of f. If $f_x > 0$ and $f_{xx} < 0$, so that f is concave in x, the effect of the Wiener process is to pull the drift down relative to the non-stochastic trend (note that the actual time path of the stochastic y will depend on the actual realizations of the Wiener process). Thus the presence of the Wiener process in x has an impact on the drift of y, but that impact will vary depending on the shape of the functional relation between x and y. In addition, even if α and f_t are zero, so there is no trend in x or in the deterministic equation for y, the drift term in the stochastic equation for y will be

(22) $[Edy]/dt = ½ f_{xx} \sigma^2$

so the presence of the Wiener process will add a drift, positive or negative depending on the sign of f_{xx}, with magnitude depending on the magnitudes of f_{xx} and of σ^2.

The fact that the presence of Wiener-type noise in x can produce drift in y even when there is no drift in x is a Jensen's Inequality effect. The upward and downward shocks to x are assumed to be identically distributed. If we assume that f(x) has the usual concave shape of a utility function, with $f_x > 0$, $f_{xx} < 0$, each upward step in x will produce an upward step in y and there will be the same number of upward as downward steps in y, but if we compare the effect of an upward step in x with that of a downward step of equal magnitude in x, the upward step in y produced by the upward step in x will be smaller than the downward step in y produced by the (equal magnitude) downward step in x. Thus over the long run, when x is hit by an equal number of identically distributed upward and downward shocks, y will have an equal number of upward and downward steps but the downward steps in y will tend to be larger than the upward steps. The result is that even though x will not have any particular drift over the long run (although it might appear to have drift in the short run if for example, it happens to be hit by a string of upward shocks), y will tend to have a downward drift over the long run.

To introduce this sort of uncertainty into our optimal control problem we adopt the approach in Pindyck (1982). This involves starting from what is essentially a dynamic programming approach to the problem and using either the standard calculus or the Ito calculus as appropriate in deriving what is in effect a stochastic Hamiltonian problem. We can use the standard calculus in taking derivatives which do not involve the Wiener process: in effect we could take second order Taylor Series expansions in place of the first order ones which the standard calculus involves, then invoke the rules of multiplication from the Ito calculus to make the second order terms vanish. In effect, the Ito calculus extends the standard calculus to the stochastic case, and if there is no stochastic element in a particular differentiation, the two approaches give the same result. Broadly speaking, Pindyck's approach has two defining characteristics. One is that it replicates the steps used in deterministic optimal control analysis, giving a stochastic equation of motion for the addictive commodity, or drift term, for the control variable, which can be compared with the equation of motion derived from the deterministic case. The other is that, unlike the dynamic programming approach, it eliminates direct reference to the value function and therefore does not require an assumption as to a functional form for the value function. General assumptions still need to be made about the functional form of the instantaneous objective function, but can, in a sense, start with these rather than making them fit an explicitly solvable form for the value function.

In the stochastic rational addiction problem we will use the same form of instantaneous utility function as in the deterministic problem:

(23) $U = U(Y - pS, S, A)$

We introduce the Wiener process by modifying the equation of motion for A:

(24) $dA = [S - \delta A]dt + \sigma dz$

In (24), the drift term in dA is the same as the time derivative of A in the non-stochastic case, and we have added a Wiener process term to the drift[7]. Here the uncertainty might be about the individual's own susceptibility to addiction and about the damage her cumulative smoking is doing to her health, while σ might represent epidemiological or population health level information about the harm done from smoking. The individual smoker might have a statistical sense of the magnitude of A but not know what her own particular value of A is at any time. This is particularly likely given the inherent unobservability of A at the individual level. When the individual is making her forward-looking smoking plan, she would presumably base her prediction about the rate at which her own A will accumulate as a result of her smoking on population health information, with the awareness that her own actual accumulation path might vary from the epidemiological norm. This unobservability of her own, individual, equation of motion for A could be regarded as the reason she treats her actual value of A as a stochastic variable. Thus we are treating the stochastic element primarily as a measure of our smoker's ignorance about her own susceptibility to addiction, and are not assuming that her own, actual, stock of addiction capital necessarily follows this particular Weiner process. Our individual knows that she is consuming an addictive good and knows that her stock of addiction will increase with the amount she smokes and with the length of time she has been a smoker. We are assuming that the public health information which is available about the addictiveness of cigarettes gives her a sense of how her own addiction can be expected to evolve over time – this expectation is the drift term in the Weiner process for A. She also knows that at the population level there is a distribution of actual levels of addiction around the expected level for any group of individuals who have been following the same cigarette consumption time paths. She might infer this, for example, from ex-smokers anecdotes about how easy or difficult they found quitting. The spread is presumably due to the existence, in the population, of a distribution of susceptibilities to addiction which are unobservable until people do try to quit. Our smoker does not know her own susceptibility to addiction, so she must assume that she will be somewhere in the population distribution. Thus the drift term in the Weiner process is a reflection of what is known about the typical smoker's tendency to become addicted and

[7] We use an additive Wiener process in our application. One technical drawback to this form is that it does allow A to become negative as a result of random shocks. This could be dealt with by making the Wiener process multiplicative rather than additive: i.e. using $A\sigma dz$ rather than σdz. Since, however, our interest is in the case of smokers who find it difficult to quit because of an unexpectedly large accumulation of A, the difference in our formal analysis will be small.

the stochastic term is a reflection of her ignorance about where she is in the population distribution of susceptibility.

Note that S does not enter the random element, so the individual cannot adjust the degree of uncertainty inherent in her stock of addiction capital directly by adjusting her smoking behaviour. All she can do is change her plan for an inter-temporal consumption pattern in the face of uncertainty. Here

(25) $EdA = [S - \delta A]dt$, so that $EdA/dt = [S - \delta A]$ and

(26) $Var(dA) = E[dA - EdA]^2 = E[\sigma dz]^2 = \sigma^2 dt$

Next, we introduce a dynamic programming-style maximum value function:

(27) $J(A,t) = Max_S\, E \int_t^T U(Y - pS, S, A) e^{-\rho\tau}\, d\tau$

Note that J is written as a function of the state variable of the problem, and of t.

We can re-write (27) by using Bellman's Principle:

(28) $J(A,t) = Max_S\, E\, [\, U(Y - pS, S, A) e^{-\rho t}\, dt + J(A + dA, t + dt)]$

then taking a second order Taylor Series expansion of the J term inside the square brackets:

(29) $J(A,t) = Max_S\, E\, [\, U(Y - pS, S, A) e^{-\rho t}\, dt + J(A,t) + J_A dA + \frac{1}{2} J_{AA} [dA]^2 + J_t dt + \frac{1}{2} J_{tt} [dt]^2\,]$

The term J(A,t) appears on each side of (29), so it can be cancelled. We can also replace dA and $[dA]^2$ using (24) and the Ito multiplication and expectation rules, noting that this means that $EdA = [S - \delta A]dt$ and that $E[dA]^2 = \sigma^2 dt$. Making these substitutions gives

(30) $0 = Max_S\, [\, U(Y - pS, S, A) e^{-\rho t}\, dt + J_A [S - \delta A] dt + \frac{\sigma^2}{2} J_{AA}\, dt + J_t dt]$

Our next step is to find the first order condition for S from (30):

(31) $[U_S - pU_C] e^{-\rho t} + J_A = 0$

Where we can use the standard calculus expressions because we are simply dealing with the derivative with respect to S – how the RHS of (30) changes when, for whatever reason, S changes, subject to the condition that the derivative must equal zero. Rewriting (31), we have

(32) $J_A = [pU_C - U_S]\, e^{-\rho t}$

Apart from the presence of the discount term on the right hand side, this is the counterpart of equation (7) above, the FOC for the current-value costate ψ in the non-stochastic case. This is because J_A is defined as the change in the value of the maximized value function J(A,t) resulting from the addition of one unit of A at time t – i.e. it has the same interpretation in the dynamic programming notation as ψ has in optimal control notation. The presence of $e^{-\rho t}$ on the RHS of (32) simply indicates that, for the moment, we are working with the maximized value function J(A,t) in present value rather than in current value terms. We will change this later in the analysis.

In the non-stochastic analysis our next step, after finding (7), was to differentiate both sides of (7) with respect to time. We do the same thing here, but in the stochastic case we must use the Ito time derivative, giving:

(33) $\frac{1}{dt}EdJ_A = -\rho[pU_C - U_S]e^{-\rho t} + e^{-\rho t}\frac{1}{dt}Ed[pU_C - U_S]$

For the moment we will leave (33) in this form and not expand the RHS Ito derivative until a later stage of the analysis.

Next, following Pindyck[8], we return to the maximized version of (30), which means that we can drop the Max_S operator, and differentiate with respect to A:

(34) $0 = U_A e^{-\rho t}\, dt - \delta J_A\, dt + J_{AA}[S - \delta A]dt + J_{tA}dt + \frac{\sigma^2}{2}J_{AAA}dt$

Dividing through by dt, we see that the last three terms on the RHS of (34) taken together are actually the expression Ito derivative of J_A(A,t) with respect to time, so we can re-write (34) as

(35) $0 = U_A e^{-\rho t} - \delta J_A + \frac{1}{dt}EdJ_A$

Rearranging this expression gives

(36) $\frac{1}{dt}EdJ_A = \delta J_A - U_A e^{-\rho t}$

[8] In this step we are making use, first, of the fact that the equality in (30) must always hold, so any changes on the RHS as a result of changes in A, must have a net effect of zero, and second of the fact that we are working with a maximum value function. This latter allows us to invoke the envelope theorem when differentiating with respect to A, which is why no S_A terms appear in (34), even though the optimized S will be a function of A, which we will use below.

Equation (32) above gave us an expression for J_A from the FOC with respect to S, so we can substitute that expression into (36) giving:

(37) $$\frac{1}{dt}EdJ_A = \delta[pU_C - U_S]e^{-\rho t} - U_A e^{-\rho t}$$

We now have two expressions for the Ito derivative of J_A, equations (37) and (33), so we can equate these, giving

(38) $$-\rho[pU_C - U_S]e^{-\rho t} + e^{-\rho t}\frac{1}{dt}Ed[pU_C - U_S] = \delta[pU_C - U_S]e^{-\rho t} - U_A e^{-\rho t}$$

In (38) cancelling out the $e^{-\rho t}$ terms on both sides, and rearranging so that the remaining Ito derivative is isolated on the LHS, giving

(39) $$\frac{1}{dt}Ed[pU_C - U_S] = [\delta + \rho][pU_C - U_S] - U_A$$

Note that in (39) we have been able to combine two terms which involve $[pU_C - U_S]$.

Equation (39), when we have evaluated the Ito derivative on the LHS, will give the stochastic counterpart of Equation (10) in the non-stochastic problem[9].

Our next step is to evaluate the Ito derivative on the LHS of (39). Note that $pU_C - U_S$ is, with all of its arguments included, $pU_C(Y - pS, S, A) - U_S(Y\text{-}pS, S, A)$. For simplicity, for the moment, we write this as F(S,A). We will revert to the full notation after we determine the general form of the Ito derivative of F(S,A).

We begin by using a second order Taylor Series Expansion of F(S,A) at the point of expansion (S,A) to give us an expression for dF(S,A):

(40) $$dF(S,A) = F_S dS + \tfrac{1}{2} F_{SS}[dS]^2 + F_A dA + \tfrac{1}{2} F_{AA}[dA]^2 + F_{SA}dSdA$$

[9] Note that we are not trying to solve for the value function nor, as is done in many dynamic programming problems, are we trying to solve for an explicit policy function S(A). As we noted above we are assuming that A is fundamentally unobservable, so such a policy function would be infeasible. By following Pindyck's approach, we will wind up with an equation of motion for S which takes account of the drift in A, which we have identified with a population average dynamic, and the σ term, which is our smoker's assessment of her own ignorance about her own tendency to become addicted.

Since we will need an Ito derivative, we write (40) as

(41)

$$\frac{1}{dt}EdF(S,A) =$$
$$F_S\frac{1}{dt}EdS + \frac{F_{SS}}{2}\frac{1}{dt}E[dS]^2 + F_A\frac{1}{dt}\,EdA + \frac{F_{AA}}{2}\frac{1}{dt}E[dA]^2 + F_{SA}\frac{1}{dt}\,E[dSdA]$$

Next we convert the F(S,A) notation back to utility terms giving, for the partial derivatives of F:

(42a) $F_S = -p^2U_{CC} – U_{SS}$

(42b) $F_{SS} = p^3U_{CCC} – U_{SSS}$

(42c) $F_{SA} = -U_{SSA}$

(42d) $F_A = - U_{SA}$

(42e) $F_{AA} = -U_{SAA}$

(42f) $F_{AS} = -U_{SAS}$

Where we have made use of the assumptions set out in equation (1) above about which cross partials we are setting to zero.

Substituting in (41) gives

(43) $$\frac{1}{dt}EdF(S,A) = -[p^2U_{CC} + U_{SS}]\frac{1}{dt}EdS + \frac{[p^3U_{CCC}-U_{SSS}]}{2}\frac{1}{dt}E[dS]^2 + -U_{SA}[S - \delta A] - \frac{U_{SAA}\sigma^2}{2} - U_{SSA}\frac{1}{dt}\,E[dSdA]$$

Where we have also substituted for dA and $[dA]^2$. From (39), the expression in (43) is equal to

$[\delta+\rho][pU_C – U_S] – U_A$, so, combining these two expressions and rearranging:

(44) $$-[p^2U_{CC} + U_{SS}]\frac{1}{dt}EdS + \frac{[p^3U_{CCC}-U_{SSS}]}{2}\frac{1}{dt}E[dS]^2 = [\delta + \rho][pU_C - U_{\square}] - U_A + U_{SA}[S - \delta A] + \frac{U_{SAA}\sigma^2}{2} + U_{SSA}\frac{1}{dt}\,E[dSdA]$$

We note here that many of the terms in (44) are common to (10), although in the Ito notation. We also note that there are still some terms which need to be expanded.

At this point in the problem, the choice variable S can be regarded as a function of the state variable A: S = S(A). Taking the Ito derivative of S(A) (since A is stochastic and therefore S will be stochastic) we have, as the second order Taylor Series expansion:

(45) $dS = S_A dA + ½\, S_{AA}[dA]^2$

From (45) we observe that $[dS]^2$, which we need for the left hand side of (44), is

(46) $[dS]^2 = S_A{}^2[dA]^2 + ¼\, [S_{AA}{}^2][dA]^4 + S_A S_{AA}[dA]^3$

Of which the only term on the RHS which will survive when we apply the rules of multiplication will be $S_A{}^2[dA]^2$, giving

(47) $[dS]^2 = S_A{}^2\sigma^2 dt$

On the right hand side of (44) we see that we will need the term dSdA, which will be $S_A\sigma^2 dt$. Substituting into (44) and rearranging gives:

(48) $-[p^2U_{CC} + U_{SS}]\frac{1}{dt}EdS = [\delta + \rho][pU_C - U_S] - U_A + U_{SA}[S - \delta A] + \frac{U_{SAA}\sigma^2}{2} - \frac{[p^3U_{CCC} - U_{SSS}]S_A{}^2\sigma^2}{2} + U_{SSA}S_A\sigma^2$

Finally, isolating the Ito derivative of S on the LHS of (48) gives

(49) $\frac{1}{dt}EdS = \frac{[\delta+\rho][U_S - pU_C] + U_A - U_{SA}[S - \delta A]}{[p^2U_{CC} + U_{SS}]} + \frac{[p^3U_{CCC} - U_{SSS}]}{[p^2U_{CC} + U_{SS}]}\frac{S_A{}^2\sigma^2}{2} - \frac{[2U_{SSA}S_A + U_{SAA}][\sigma^2/2]}{[p^2U_{CC} + U_{SS}]}$

The first term on the RHS of (49) is the RHS of (11), the equation of motion for S in the non-stochastic case. The remaining two terms on the RHS are introduced by the stochastic nature of the problem, as can be seen from the fact that both of the new terms vanish if σ is set equal to zero. We see here, as we suggested when we introduced the Wiener process into A, that the individual is responding to the presence of widening uncertainty about future levels of A by adjusting her expected, or planned smoking trajectory. She cannot affect σ directly through her choices of S, so she responds by, in effect, tilting her smoking path.

One thing which is immediately obvious is that the stochastic terms, which will make the drift in the optimal time path of S differ from the non-stochastic trajectory, depend on third derivatives of the utility function. This means that, if we adopt the assumption of a quadratic utility function, as, following Becker and Murphy, is commonly assumed in the RA literature, we are imposing the conclusion that Wiener-type uncertainty about the accumulation of addiction capital should have no effect on our consumer's lifetime consumption trajectory of S[10]. This is not, however, the limit of our ability to give some interpretation to the effects of the introduction of the stochastic element to our individual's optimal consumption of S.

Consider the first of the new RHS terms, involving the own third partials of the utility function. To get a sense of what these high-order derivatives mean for individual behaviour, note that, under our assumption that the budget constraint is binding at each instant, the individual's instantaneous utility can be written as a function of S and A: V(S,A). In a 1990 Econometrica paper, Kimball (1990), working with the utility function U(C), defined the term $-U_{CCC}/U_{CC}$ as the coefficient of absolute prudence. Kimball regarded this as the counterpart of the coefficient of absolute risk aversion, $-U_{CC}/U_{CC}$. In our case we can define $-V_{SSS}/V_{SS}$ as the coefficient of prudence in the consumption of S. The logic of Kimball's definition rests on the fact that we are interested in solving for the optimal level of the choice variable(s) and in looking at how the introduction of risk, or an increase in risk, would affect the optimal level of the choice variable. Figure 2 below sketches out a generic case, where our interest is in the choice variable on the horizontal axis.

The optimal level of a choice variable depends not on the level of the individual's utility function but on her marginal utility function – the optimal level will always involve some sort of equating of marginal benefit and marginal cost. Thus if we are interested in looking at how the introduction of uncertainty affects the optimal level of the choice variable, we need to consider not so much the curvature of the utility function itself as the curvature of the marginal utility function. Since the marginal utility function is the first derivative of the utility function, its curvature, which involves its second derivative, will be written in terms of the third derivative of the original utility function.

Eeckhoudt (2012) notes that for the most part after Kimball's introduction of the concept of measurable prudence, its appearance in the literature tended to be restricted to the literature on precautionary saving. Since the mid-2000s, however, Eeckhoudt notes, it has increasingly been used in a broader range of literature.

In our application, if we define the coefficient of prudence with regard to S as $-V_{SSS}/V_{SS}$ and note that $V(S,A) = U(Y-pS, S, A)$ then $V_S = U_S - pU_C$, $V_{SS} = U_{SS} + p^2U_{CC}$ and $V_{SSS} = U_{SSS} - p^3U_{CCC}$. Then

(50) $-V_{SSS}/V_{SS} = -[U_{SSS} - p^3U_{CCC}]/[U_{SS} + p^2U_{CC}] = [p^3U_{CCC} - U_{SSS}]/[U_{SS} + p^2U_{CC}]$

Which is the first of our two stochastic terms on the RHS of (49). The sign of this term is indeterminate at this point, since it depends on the signs and magnitudes of two third derivatives of the original utility function. The usual assumption in the risk literature is that the derivatives of utility functions alternate in sign, so that these two third derivatives would both

[10] With a quadratic utility function, the individual's level of utility will be affected by the uncertainty but she will not adjust her consumption in response to it.

be positive. (We clearly cannot make this assumption to casually, as we have already assumed that the first two derivatives of U with respect to A, U_A and U_{AA} are both negative, since A is a bad rather than a good). Note that the presence of the U_{CCC} term reflects the fact that we have assumed that the budget constraint is binding at each instant of time so that an increase in S immediately results in a reduction in C. If we extend the analysis to involve a lifetime budget constraint rather than an always-binding instantaneous constraint, an increase in S would probably result in some reduction in C in the same period, but the effect might also be spread over several periods. The prudence term is multiplied by $S_A{}^2\sigma^2/2$, so the impact of the direct prudence effect on the optimal level of S at each period depends on the magnitude of the uncertainty which has been introduced and on the magnitude of the optimal response of S to A but not on the sign of that response, since S_A enters squared. The sign of the overall effect, however, depends on the balance of the curvature of marginal utility in the S direction and marginal utility in the C direction.

The second of the stochastic terms on the RHS of (49) depends on what we might regard as a cross-prudence effect. Here we can isolate an effect which is unique to the RA literature. The term U_{SA}, which we have assumed is positive, reflects the strength of the complementarity between S and A or, in RA terms, the strength of the addictiveness of A since it reflects the degree to which an increase in the individual's accumulated addiction capital increases the marginal utility which she derives from S – from smoking, for example – and hence how her consumption choices will shift in favour of smoking. Looking at the last term on the RHS of (49) we note that it depends on U_{SAA} and U_{SSA}. Since order of differentiation does not matter, we can write these as U_{SAA} and U_{SAS}, and note that they reflect how changes in the levels of A and S respectively affect the complementarity, or strength of addiction term, U_{SA}. We also note that, while this term depends on σ^2, the optimal consumption policy function enters through $S_A(A)$, not through the square of that term as in the previous stochastic term. Thus the sign of the individual's response in terms of smoking intensity to an increase in her stock of addiction enters into the determination of the optimal consumption trajectory. As a corollary we note that if the commodity is harmful but not addictive, as in the case of the model developed by Ippolito (1981), so that $U_{SA} = 0$ for all S and A, the last term vanishes. The first stochastic term on the RHS of (49), however, will not vanish, so the magnitude of the optimal response of S to A will enter under conditions of uncertainty.

We can translate the last term on the RHS of (49) into Kimball's terms if we note that U_{SAA} can also be written as U_{AAS} in which case the two component terms are U_{SSA} and U_{AAS}. Then we can think of the first of these as showing how an increase in the level of A changes the curvature of the marginal utility of S curve, and the second as showing how an increase in S changes the curvature of the marginal disutility of A term, noting again that we have assumed that U_A and U_{AA} are both negative, so we need to be careful in thinking about the curvature of the latter.

One thing which is immediately obvious about equation (49) is that there is nothing immediately obvious about exactly how the introduction of Wiener-type uncertainty affects the individual's optimal consumption of an addictive commodity at any point. We can, however, make a start at evaluating this by noting that the first term on the RHS of (49) is the $\dot{S}$ expression from the non-stochastic problem. If this term is set to zero, as it will be on the stationary locus on the phase diagram for the non-stochastic case, we could, if we could sign the remaining two terms, judge whether in the stochastic model S would be increasing or decreasing in expectation at (S,A) values which would make it stationary in the non-stochastic model. This would, however, require us to be able to say more than we can at present about the signs of the two stochastic terms in (49).

In terms of guidance for empirical work, we can make a few points with respect to equation (49). One is that in empirical implementation, it might well make sense to specify the dependent variable in first difference form, rather than estimate a difference equation in levels. A second, emphasized by the presence of the S_A terms on the RHS of (49), is that we can expect the appropriate functional form of an equation relating changes in S to the level of a will be non-linear. This is emphasized by the third derivative, prudence, terms on the RHS. These terms can themselves be expected to change as a single individual moves along her optimal lifetime (S,A) trajectory, adding to the tendency to non-linearity. In addition, since although our theoretical model is in terms of a single optimizing individual, any empirical implementation will result in what is in effect an average of the behaviours of a large number of individuals, we should note that there is likely to be a range of values of the prudence terms across individuals, meaning that we will have to give consideration to aggregation issues. It seems unlikely that representative agent modelling will be able to take us very far in empirical implementation of our model.

IV. Conclusion

In this paper we have attempted to show how the introduction of a particular type of uncertainty with regards to the smoker's lifetime accumulation of addiction capital might affect her rational long-term smoking plan. While we have not derived an explicit functional form for S, we have, by adopting Pindyck's (1982) approach, shown what additional considerations arise when we introduce uncertainty, noting in particular that we need to give careful thought to the use of estimation forms derived using the assumption of a quadratic utility function. It may be that future empirical work in RA will need to relax the assumption that S is a linear function of its own lead and lag values, and test more flexible functional forms. We have not, in this paper, tried to move from a system of two first-order differential equations, one in S and one in (the typically unobservable) A, to a single second order differential equation in the single observable variable S. Our results in equation (49) may, however, provide some guidance as to how we should advance empirical research into the consumption of addictive commodities.

One point to recall is that, unlike in other cases of stochastic state variables – models in the finance literature in which the return on assets is stochastic, for example – the unobservability of A means that our smoker cannot be assumed to be adjusting her level of S on a continuous basis as observations about current values of A come in. Her optimal expected-utility maximizing trajectory therefore is likely to be, or be very close to, her actual smoking trajectory. In a very real sense she will not know how addicted she is until she reaches the point at which she planned to quit. Our rational smoker will have adjusted her smoking trajectory in recognition of the unobservability of A, on the basis of population health information about σ, but she cannot eliminate σ altogether. Thus even a fully rational smoker, who follows the trajectory designed to maximize her lifetime expected utility, may find it much more difficult to quit than she had hoped. In particular, in contrast to much of the dynamic programming literature, we cannot define a policy rule of the general form S(A(t)) which will allow our smoker to adjust her current smoking habits in response to her current level of addiction capital[11]. She will make her smoking plan at time 0, presumably tipping its trend down to allow for uncertainty and unobservability, and follow it until she reaches the time at which she had planned to quit. If her stock of A at that time is less than she had expected, she will find quitting to be easy, but if it is significantly greater, even a rational smoker might find quitting to be difficult.

One implication of the unobservability of A may be that we should reconsider replacing the common lead-lag second-order difference equation form used in empirical RA work by a backward-looking second order difference equation. Since the individual's actual value of A, though unobservable, will be a function of their cumulative past smoking, estimation based on equation (49) might do a better job of catching the individual's beliefs about their current level of A (which will determine their current and future smoking behaviour, according to (49)) if it included several periods worth their known past smoking levels than if it includes their future S. We should also, perhaps, pay more attention to functional form, since the right hand side of (49) can be expected to be non-linear[12].

Perhaps the most important conclusion, however, pertains to the use of the quadratic for of the utility function. This assumption had advantages, in that it makes the marginal utility functions linear, but the benefits of this simplification might have carried too heavy a price[13]. Under the assumption of a quadratic utility function, Wiener-type uncertainty will affect the individual's level of utility, but she will not make any changes to her planned consumption trajectory to allow for the presence of that uncertainty. This seems unlikely, if our smoker is truly a rational consumer of an addictive commodity. This implication might also provide insights into the well

[11] In the dynamic portfolio literature, for example, she could presumably adjust the weights on the assets in her portfolio depending on how the actual value of her portfolio had evolved.

[12] We note that in estimating an equation based on (49), we still need either to make assumptions about the form of the utility function or to adopt some kind of flexible functional form. The advantage of leaving this choice until the estimation stage instead of making an assumption about the functional form of the J(K,t) maximum value function early on is that it allows us to see clearly the role of the third derivatives of the instantaneous Utility function in the theoretical and empirical analysis.

[13] This is not unique to our stochastic application. The risk aversion literature also raises serious doubts about the value of the quadratic simplification of the utility function.

known[14] fact that empirical rational addiction analyses frequently come up with frankly absurd estimates of the individual's discount rate. These estimates are based on the ratio of the coefficients on lead and lag consumption levels in a linear equation derived from the assumption of a quadratic utility function. If rational smokers are tilting the time slopes of their cigarette consumption trajectories, in response to their assumption that they face Wiener-type uncertainty with regards to A, those coefficient ratios might in fact combine pure time preference with a response to the stochastic nature of the optimization problem, and the interpretation of the ratio in terms of the rate of pure time preference alone might be a misspecification.

Indeed, as we noted above, imposing the assumption of a quadratic utility function means that we are assuming that while the consumer's utility is affected by the stochastic nature of her problem, and she is aware of this, she does not change her consumption plans, relative to the non-stochastic case, in response to the uncertainty. This seems unlikely, especially if we seriously believe that she is rational. Perhaps the reason for some of the odd estimates reported in the empirical RA literature is that smokers are more rational than we are assuming them to be.

[14] See, for example, Laporte et al. (2016).

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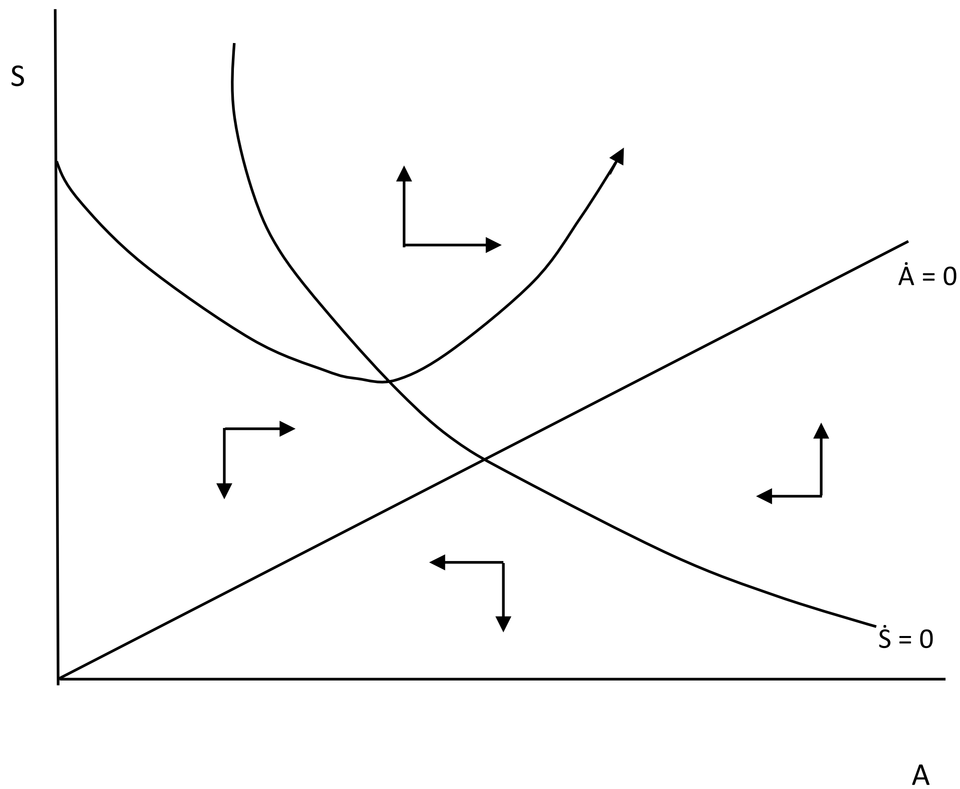

Figure 1: Phase Diagram, Deterministic Case

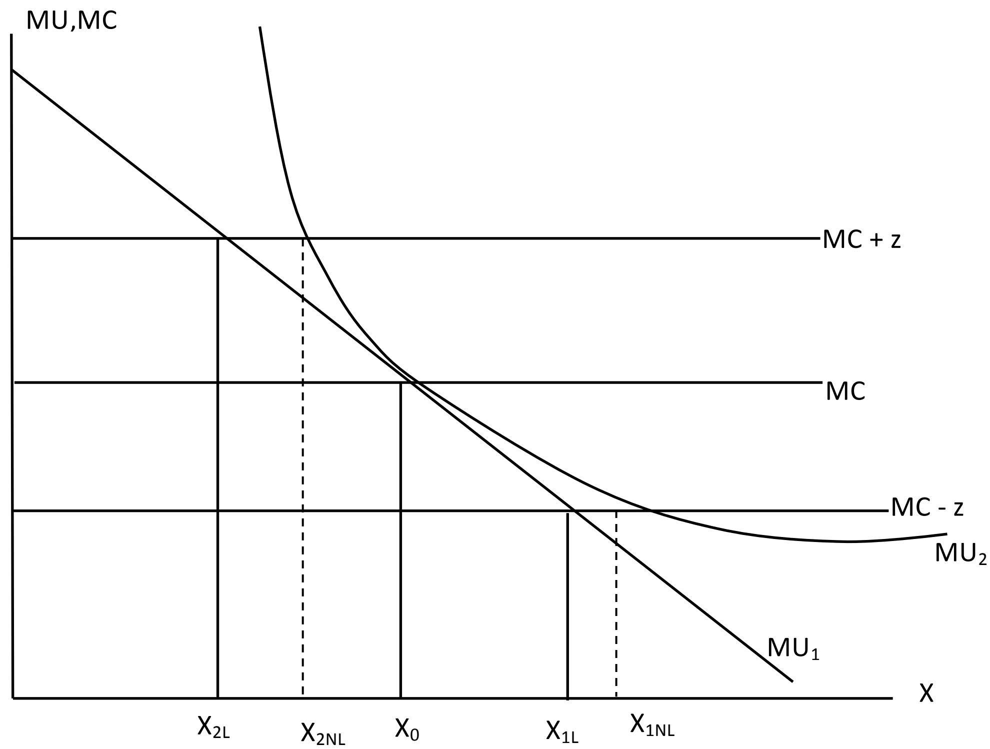

Figure 2: Prudence Effect – curvature of MU.