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On a Possible Problem in the Estimation of Saddle-point Dynamic Economic Models

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I Introduction

It is probably fair to say that econometric dynamics has devoted considerably more attention to the cases of stable and unit root dynamic behaviour than it has to the case of unstable dynamic behavior. This does not mean that the statistical analysis of unstable roots has been neglected completely (see White (1958), Anderson (1959), Chan and Wei (1988)), but that the emphasis has been on the econometrics of dynamics driven by roots on and within the unit circle[2].

Clearly, this distribution of effort arises from the fact that most economic models are dynamically stable. Even unit root macroeconomic models have stable dynamics underlying them: when the dependent variable exhibits unit root behavior it is because it has inherited it from the explanatory variables - once that effect has been properly controlled for, the error correction mechanism involves a stable dynamic process by which the dependent variable converges to its long run value.

There exists, however, an important class of theoretical models for which unstable dynamics are a fundamental part of the solution behavior. This class consists of those models whose solutions involve saddle-point dynamics. In a typical saddle-point model, the dynamic behavior of the system when it is not at its equilibrium point is driven by two characteristic roots, one stable and one unstable. These models are increasingly popular in the applied econometrics and economics literatures to for example, characterize such things as optimal asset accumulation or human capital accumulation. In this paper we consider certain issues that arise in the empirical estimation of the transition dynamics of such models.

II Theoretical Background

The class of problems with which we are interested here is that of intertemporal optimization, or optimal control, problems. Optimal control problems can be set up either in discrete or continuous time terms (see, for example, Chow (1999), Leonard and Long (1992), Ferguson and Lim (1998, 2003)) although for empirical purposes we will want to convert to discrete time terms. We can write a generic optimal control problem as

$$Max \int_0^T U(c,S)e^{-\rho t}dt \tag{1}$$

Subject to

$$\dot{S} = g(c,S) - h(S) \tag{2}$$

where c is the control variable for the problem and S the state variable. S is in essence a capital variable that accumulates and de-cumulates over time according to the first order differential equation (2). We can think of g(c,S) as a production function for gross S and h(S) as the depreciation function for S.

[2]For papers dealing with explosive behavior in unit root macro models, see Nielsen (2001), Nielsen and Reade (2004) and Juselius and Mladenovic (2002).

Combining (1) and (2) into the Hamiltonian for the problem gives

$$\mathcal{H} = U(c, S) + \psi[g(c, S) - h(S)] \tag{3}$$

where ψ is the co-state variable for the problem - the shadow price of S.

Solving the necessary conditions for the problem and making the appropriate substitutions, we can derive the phase diagram for the problem which, in generic terms in state-control space, typically looks something like Figure 1 below:

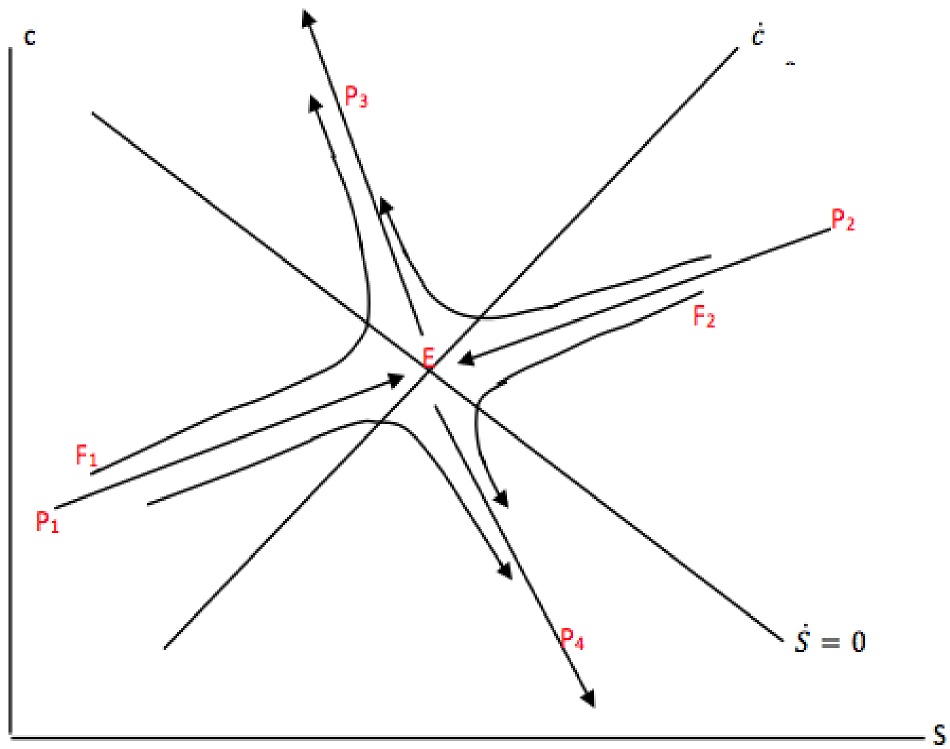

Figure 1: Phase Diagram

In Figure 1, the line $\dot{c} = 0$ is the stationary locus for the control variable, c while the line labelled $\dot{S}$ = 0 is the stationary locus for the state variable, S. The stationary loci serve to divide state-control space into regions in which S and c are increasing or decreasing. At E, the intersection of the two stationary loci, neither S nor c displays any intrinsic tendency to change. This means that if the system happens to be at point E, neither variable shows any intrinsic tendency to move away from it, making E the equilibrium for the system.

In a dynamic optimization problem the equilibrium point is generally of less interest than the transition trajectories, which are represented by the other lines on Figure 1. All of the trajectories on the phase diagram satisfy the dynamics and also satisfy the necessary conditions for optimality, so each of the lines is a candidate to be the solution trajectory for the inter-temporal optimization problem at hand. Only one of them, in general, will be optimal, with the selection of the optimal trajectory depending on the initial value of the state variable, S(0), and the horizon over which the problem is to be solved.

Phase diagrams of the kind shown in Figure 1 are familiar from both macroeconomic and microeconomic dynamic optimization problems. In macroeconomics there are a range of applications, including open economy models and models of optimal economic growth (see Azariadis (1993) for several illustrations). In microeconomics, phase diagrams are used in models of optimal savings and asset accumulation, models of the accumulation of human capital and, in related areas, Grossman's (1972) model of the accumulation of health capital and the Ippolito (1981) and Becker-Murphy (1988) models of the optimal consumption of harmful and addictive commodities.

The most important technical difference between the microeconomic and macroeconomic applications lies in the horizons of the two types of problem. Macroeconomic problems tend to be infinite horizon problems - the neoclassical model of economic growth probably being the canonical case. Microeconomic problems are much more likely to be finite horizon problems, for example, an individual's life span - in some cases the horizon itself may be a choice variable, but even then it is likely to be a finite horizon problem.

In terms of Figure 1 above, the difference in the length of the planning horizon plays into the choice of optimal trajectory. The transversality condition for an infinite horizon problem is generally summarized by saying that in an infinite horizon problem the optimal trajectory is one of the stable branches - i.e. one of the trajectories leading to point E in Figure 1. If the initial value of S is small, as is generally assumed in optimal growth models, the optimal trajectory will be the one labelled P_1, meaning that, given the initial value of S, the planner selects the value of c which lies on P_1 directly above S. If the initial value of S is large, the optimal path involves picking a value of c which places the system on P_2, directly above the initial value of S. In either case, the system will tend to converge on the equilibrium, and will take an infinite amount of time to reach E.

In the case of a finite horizon problem, as is likely to be encountered in a microeconomic problem, however, optimality does not involve converging to the equilibrium (which cannot be reached in finite time in any event). For finite horizon problems the transversality condition typically takes on one of three possibilities: either we have a fixed endpoint problem, in which case a terminal value of S is specified, or, if the endpoint is free, optimality will tend either to require that S reach 0 at precisely T, the end time for the planning problem, or the co-state variable ψ must equal 0 at time T, a condition which can usually be translated into a condition on the terminal value of the control variable c. In the finite lifetime financial asset accumulation problem, for example, a predetermined bequest target would impose a fixed endpoint value on financial assets S, and the absence of any bequest motive will typically mean that optimality involves running financial assets to 0 as the end of the individual's life is reached. These types of problems have optimal trajectories like F_1 or F_2 in Figure 1 above. (For an application to the Becker-Murphy model see Ferguson (2000).)

Formally, Figure 1 represents what is known as saddle-point dynamics. Our focus in this paper is on certain issues that arise when we are testing the hypothesis that our data set is derived from an intertemporal optimization model which yields saddle-point dynamics.

The solution to such a theoretical model can be written (in discrete time notation) as:

$$\begin{bmatrix} c_t \\ S_t \end{bmatrix} = \begin{bmatrix} \gamma_{cc} & \gamma_{cS} \\ \gamma_{Sc} & \gamma_{SS} \end{bmatrix} \begin{bmatrix} c_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} \delta_{c1} & \delta_{c2} \\ \delta_{S1} & \delta_{S2} \end{bmatrix} \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} \tag{4}$$

where the X variables include the constant and exogenous explanatory variables and the disturbance term. For our purposes in this paper we will assume away exogenous explanatory variables other than the constant. The assorted X variables determine the location of the equilibrium point, E, in Figure 1 above, and changes in those variables shift the location of the stationary loci for c and S and hence change the location of the equilibrium. In the simulations we run below, the fact that we have assumed away X variables means that the equilibrium point does not change. Making this assumption allows us to focus on a single issue relating to the dynamic behavior of the adjustment trajectory.

In the theoretical dynamics literature, two results are derived from a solution system of the form (4) (see Ferguson and Lim (1998, 2003)). The first concerns the roots of the system. The dynamics of a two-variable system such as (4) are determined by two characteristic roots, λ_1 and λ_2. Since these roots are the characteristic roots of the first matrix on the right hand side of (4), both c and S obey the same roots, although the weights on each root may differ across the two variables. Thus we can write the solution expressions for c and S from (4) as

$$c_t = A_{c1}\lambda_1^t + A_{c2}\lambda_2^t + c^* \tag{5a}$$
$$S_t = A_{S1}\lambda_1^t + A_{S2}\lambda_2^t + S^* \tag{5b}$$

where c^* and S^* are the equilibrium values of c and S. Because we are assuming that the equilibria do not shift, we do not attach a time subscript to these terms. The A_{ij} terms are constants, determined as part of the process of solving the optimization problem. In terms of Figure 1 above, these terms determine (and are determined by) which trajectory is the optimal one. The t-superscript on the roots is a time exponent, so we are writing solutions for c and S as functions of elapsed time. We take t = 0 to be the starting point for the optimization problem. We note that, while the roots, λ_1 and λ_2 are the same across (5a) and (5b) (hence their having only one subscript each), the weights, A_{ij}, can differ. The fact that the roots must be the same means that the basic nature of the dynamics of c and S must be the same (so, for example, if one were capable of cyclical behavior, the other would also have to display cyclical behavior) but the fact that the weighting terms can differ means that the trajectories of c and S will not, in general, be identical.

The fact that the roots are raised to powers of elapsed time means that we can use these solution equations to trace out trajectories of c and S over time - plots with c and or S on the vertical axis and time on the horizontal. The fact that we derive the roots from the intertemporal optimization problem underlying Figure 1 means that we can relate equations (5) to trajectories on Figure 1, as well as to time plots of c and S. One thing that is immediately clear from Figure 1 is that, while we cannot have full cycles under saddle-point dynamics, either c or S could change direction as time passes. It is also clear that, given that saddle-point behavior means that the system, and each variable in it, is driven by one stable and one unstable root (i.e., because we are working with difference equations, one λ_i less than 1 and one greater than 1), even if the unstable root (which we shall for convenience assume to be the second root) has a very small A_{ij} value, eventually, as t becomes large, λ_2^t will eventually become so large as to dominate the solution trajectory (since λ_1 is less than 1, as t grows, λ_1^t will tend to converge on zero).

Equations (5) let us talk about the difference in behavior across different types of models. As we have already noted, saddle-point dynamics do occur in macroeconomic models, but since those models have infinite time horizons the optimal trajectory is, depending on the initial value S_0, going to be one of P_1 or P_2. Clearly a trajectory which follows the stable branch cannot display explosive behavior, but at the same time the fact that we are working with a saddle-point model means that we must have one stable and one unstable root - it is the presence of the two kinds of roots which gives the solution a knife-edge character. For the system to be on one of the stable branches, the weights A_{i2}, I = c, S, must both be zero so that the unstable root plays no part in the observed dynamics, which are driven entirely by the stable root (for a related discussion, see Sengupta (1997)). Similarly, were the optimal trajectory to coincide with trajectories P_3 or P_4, the unstable branches, it would be necessary that A_{i1}, I = c, S both equal zero so that only the unstable root was operational. One implication of this conclusion is that, along the stable or unstable branches the system will behave as if it were a first order system, not a second order system. Everywhere else on the phase diagram both roots must be operational and the system must behave as a second order system.

If we consider a path like F_1 in Figure 1 above, this means that while the trajectory is moving roughly in tandem with P_1 the stable root must be dominant but, as time passes and λ_2^t becomes larger, the trajectory turns away from P_1 and moves roughly in line with P_3. Eventually λ_2^t will grow to the point where the trajectory is dominated by (although not determined exclusively by) the unstable root.

The second point about the system set out in (4) above is that, even though it is a system of two, interrelated first order difference equations, one in c and one in S, it can be reduced to a single second order difference equation in either c or S, without losing essential information about the dynamics or the equilibrium value of whichever of c or S we choose to work with. If we work with c, we have

$$c_t = \alpha_0 + \alpha_1 c_{t-1} + \alpha_2 c_{t-2} \tag{6}$$

with equilibrium point $c^* = [\alpha_0]/[1 - \alpha_1 - \alpha_2]$, where the α_i values are derived from the coefficients of (4). The equilibrium value of c in (6) will be the same as that derived from (4). Further, since this is a second order difference equation it will have two roots, which will be the same as the two roots of the system (4). Thus, if we choose, we can analyze the behavior of the system in terms of a second order difference equation in c (or equally, in terms of a second order difference equation in S). This might be a consequences of a lack of data on one of c or S; the Becker-Murphy model of rational addiction is typically estimated using a second order difference equation in c, the consumption of the addictive commodity, for lack of data on S, which in that model is a measure of the degree of the individual's addiction. The time path derived from the estimated version of (6) will be the same as the time path of c which we would find if we were working with the two-equation system.

It is fairly clear from Figure 1 that we are likely to get different estimates of the coefficients of the difference equation driving c depending on whether we have a sample drawn predominately from the region where the true optimal trajectory, F_1, say, is tracking P_1 closely as opposed to a sample drawn from the region in which it is tracking P_3, and there is nothing novel in suggesting that if we want to obtain a decent estimate of the shape of a curved trajectory we ideally want our observations to be drawn from a long span of the curve. Our interest in this paper is in the implications for estimation of an equation of the form (6) when part of the time series being analyzed may be dominated by a stable root and another part of the same time series may be dominated by an unstable root.

III Results of Time Series Monte Carlo Experiments

We tackle this issue using Monte Carlo experiments on data sets whose true data generating processes (DGPs) are second order difference equations with one stable and one unstable root. In this paper we report the results of three sets of experiments involving DGPs akin to F_1 in Figure 1 above: i.e. processes which initially converge on, but then diverge from, their equilibria. Each data set has 500 observations, and we performed 500 replications using zero-mean, normally distributed disturbance terms. The Monte Carlo recursive regressions were written in R.

Our first case is a pure time series estimation of a second order difference equation with coefficient of 2.05 on the first lag of the dependent variable and -1.05 on the second lag, giving as roots, 1.10 and 0.95. We sorted the observations so that they were initially converging on their equilibrium and then turned to diverge from it, and used recursive regression so that we could see how adding the diverging subset affected the estimated coefficients. We reserved the first 19 of our 500 observations for initialization of the recursive regressions. The estimated coefficients are shown in Figure 2 below.

In Figure 2 we see that, after a brief period of convergence, the estimated coefficients converge on the true and remain equal to the true up until about observation 175, after which they suddenly go rather dramatically wrong. After observation 176 the coefficient estimates bounce around quite actively, although it is not clear that at any time they take on values which would look unreasonable for a second order difference equation, even though they are clearly different from the true values. To get another view of the implications of this effect, in Figure 3 below we report the estimated roots which we calculate from the estimated coefficients. The true values of the roots for this case are 1.10 and 0.95. We see from Figure 3 that the unstable root is calculated accurately throughout our data series, but that the calculated value of the stable root is accurate up to roughly observation 175, after which it goes badly wrong. Thus the unstable root comes through in this example, even in the range where the stable root would be expected to dominate, but in the latter part of the data set, while the unstable root is still coming through correctly, the stable root is not - indeed, for much of our data set we find a negative value for the stable root.

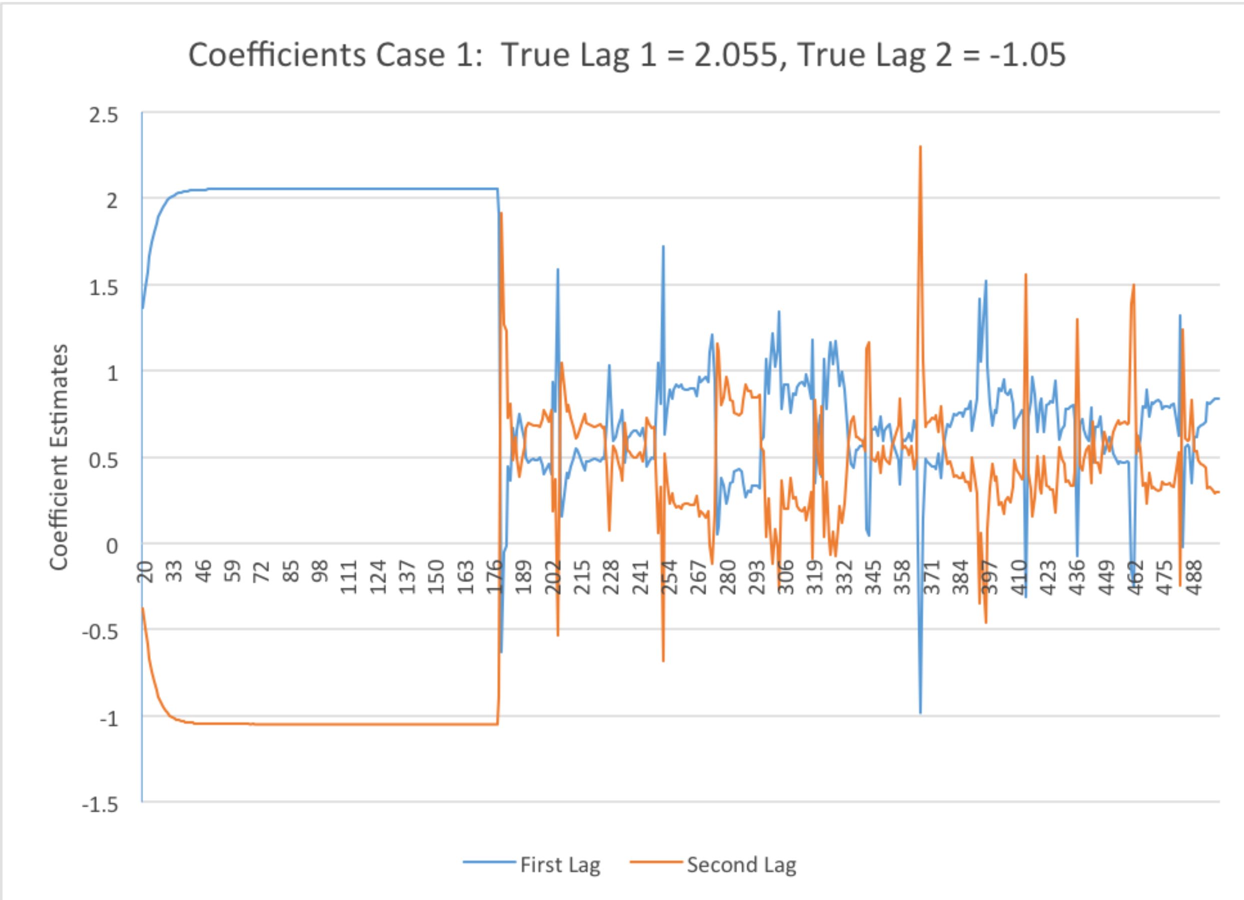

Figure 2: Means of Monte Carlo estimates of lag coefficient for Case 1

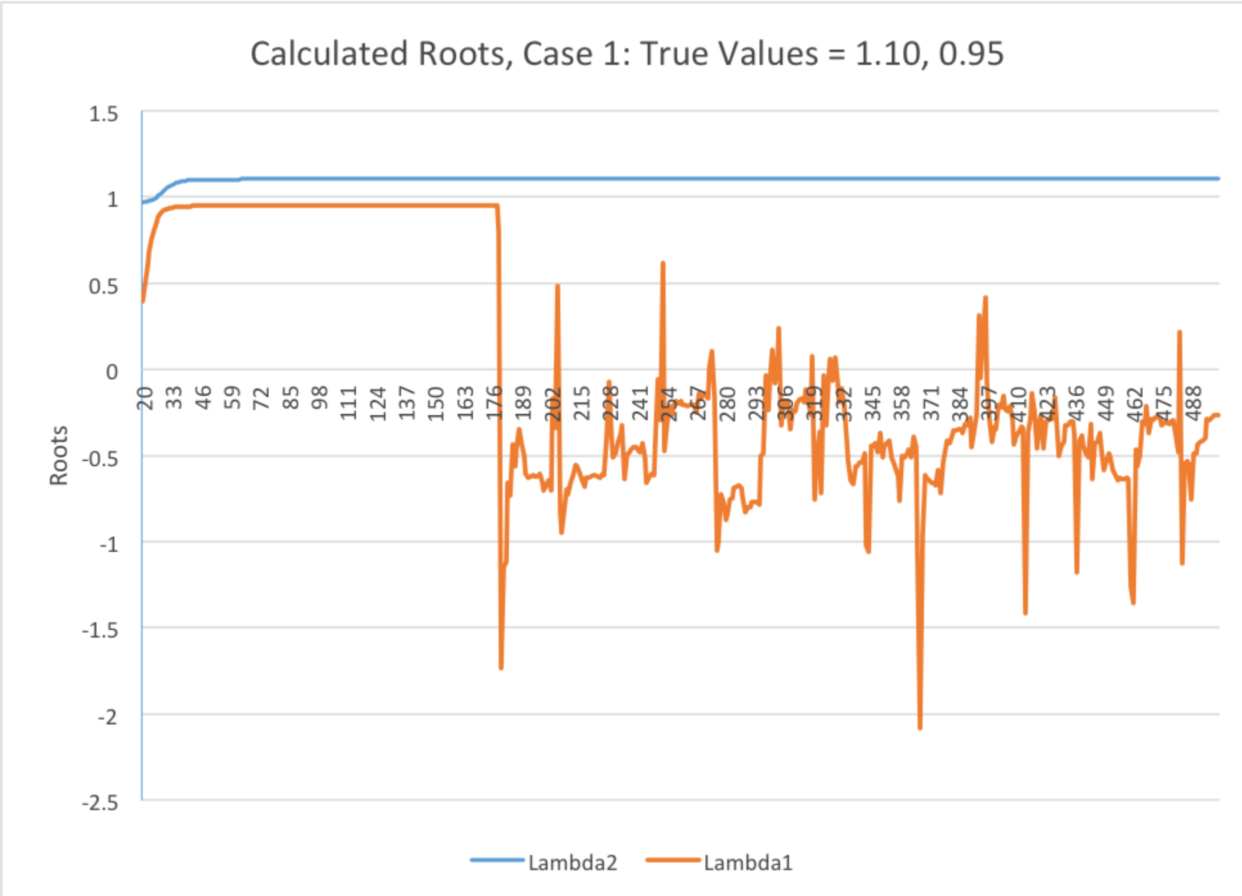

Figure 3: Means of Monte Carlo estimates of roots for Case 1

In Case 2, the true coefficients are 2.09 and -1.05, and the true roots are 1.25 and 0.84. While the coefficient values are not much different from those in our first case, one effect of the change is to shorten the well-behaved case significantly, as shown in Figure 4 below. The means of the calculated roots are shown in Figure 5 below - again, though with one blip at observation 70, the unstable root is well estimated, but the stable root is well estimated only through the well-behaved range and badly estimated, and generally negative, through the larger part of the range.

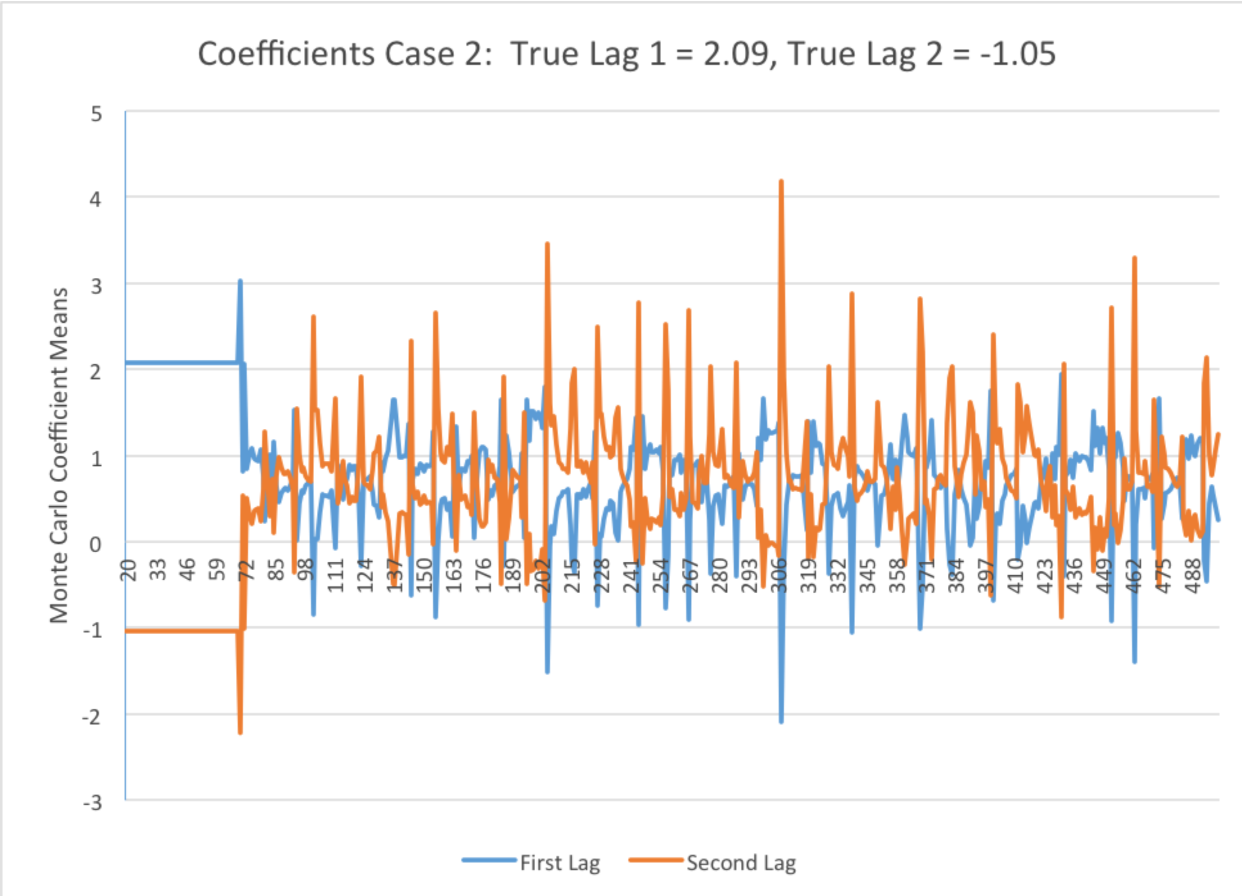

Figure 4: Means of Monte Carlo estimates of lag coefficients, Case 2

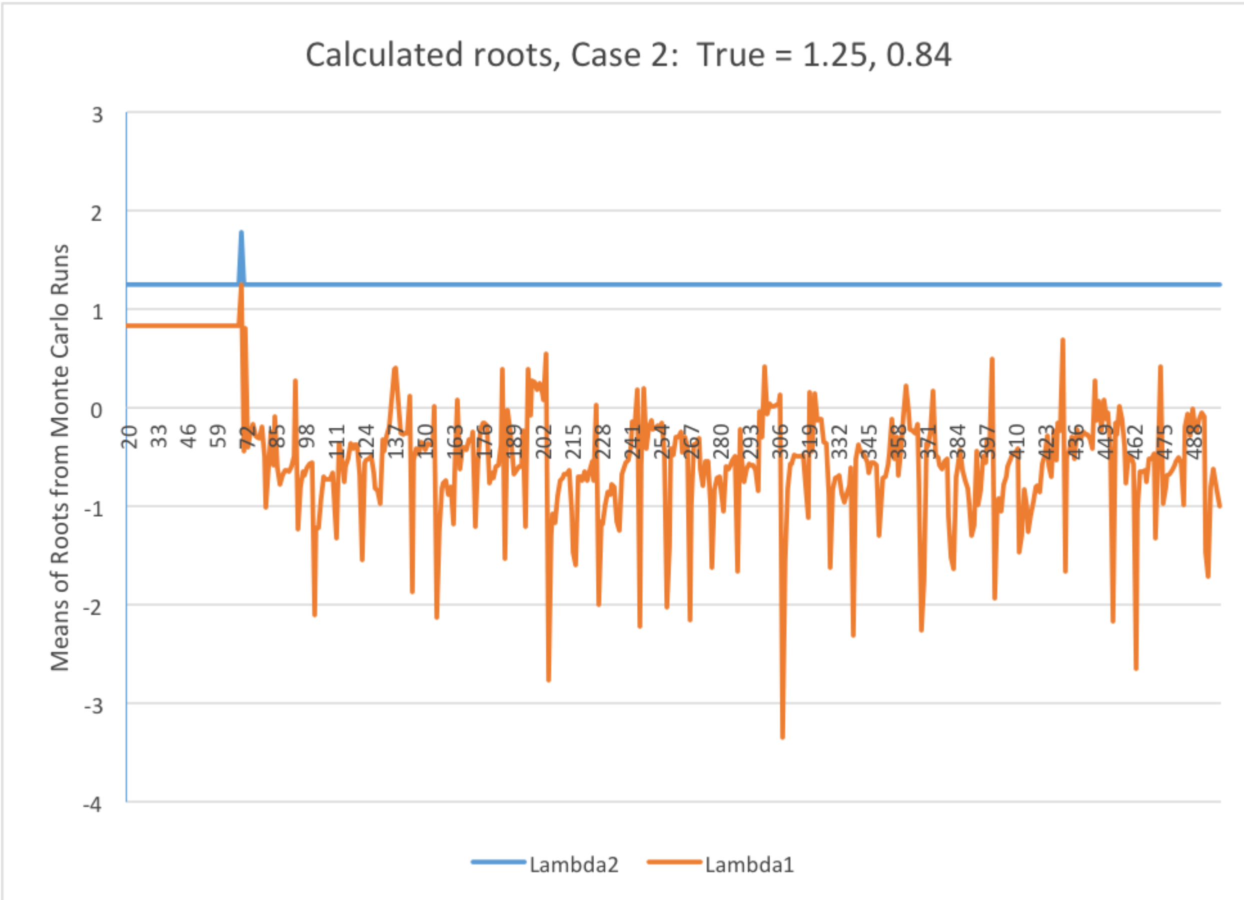

Figure 5: Means of Monte Carlo Calculated Roots, Case 2

Figure 6 below shows the Monte Carlo coefficient means for a case with true values 1.45 and -0.42. This time the well-behaved segment is much larger than in the previous cases, but the basic pattern repeats itself. The roots for this case are 1.05 and 0.40, and the Monte Carlo means, which are shown in Figure 7, show the same general pattern of matching the true through the well-behaved region, although with a couple of blips where the mean of the calculated discriminant of the roots becomes very slightly negative, giving complex roots, which are not shown in Figure 7.

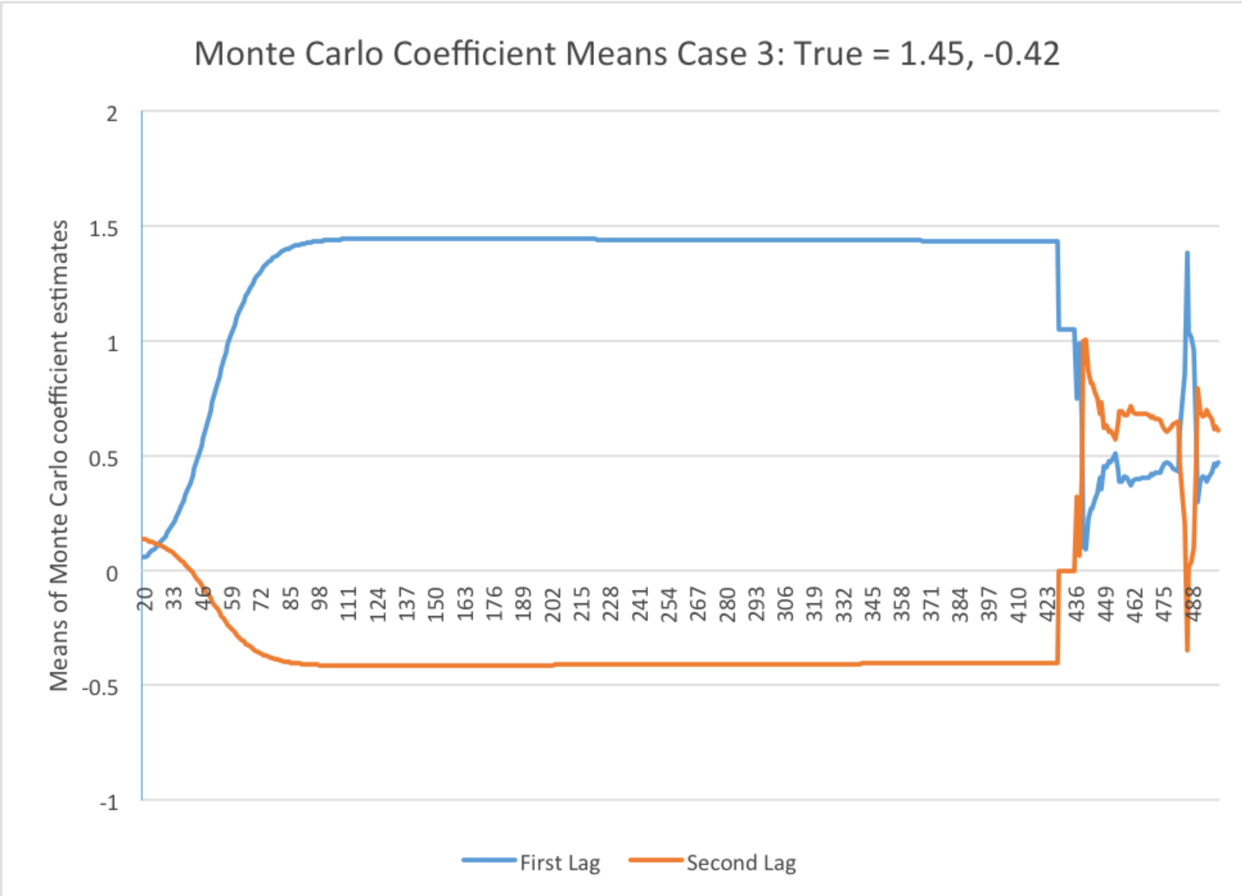

Figure 6: Means of Monte Carlo coefficient estimates, Case 3

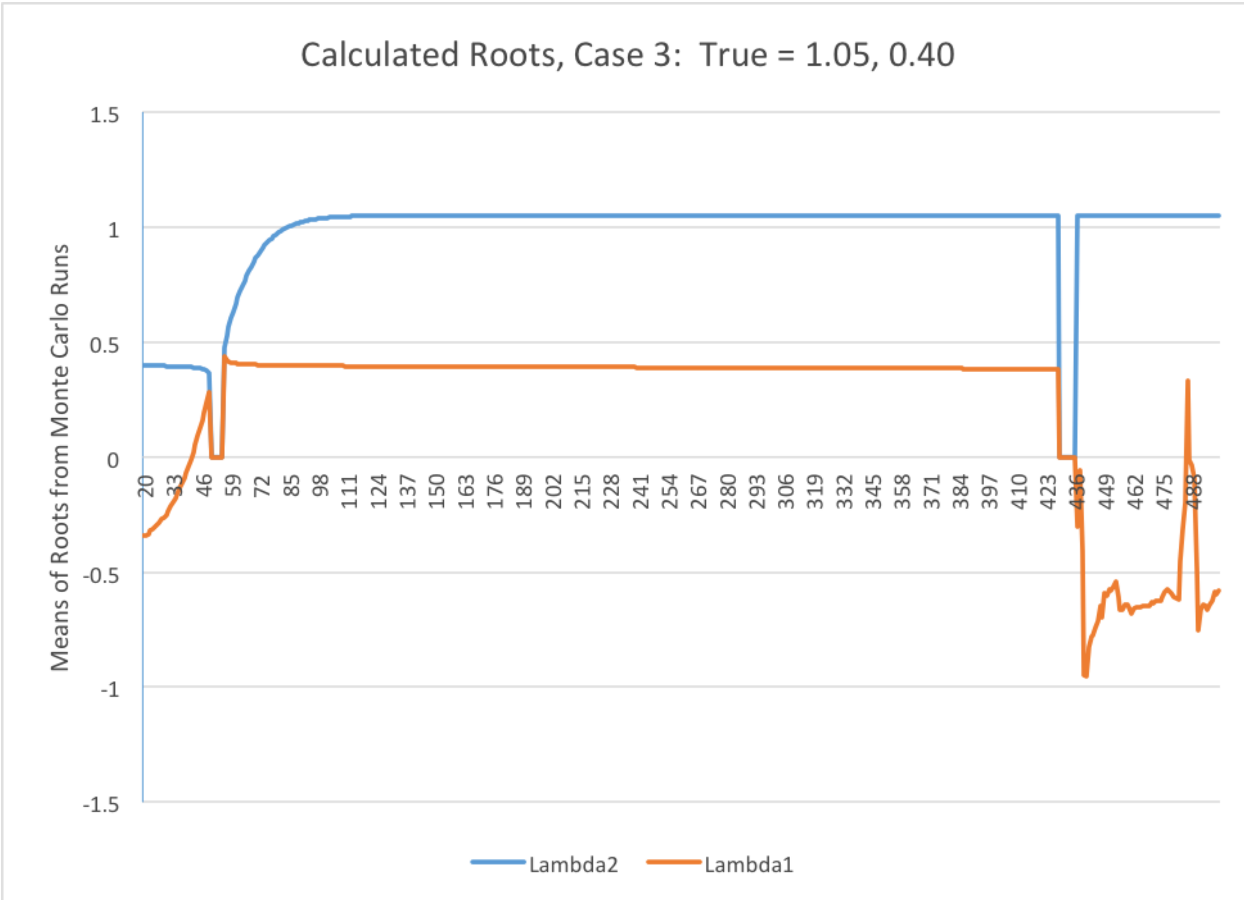

Figure 7: Calculated Roots, Case 3

IV Results of Panel Monte Carlo Experiments

In practice, of course, we will never have individual level time series of 500 observations. Even if we assume that we are looking at monthly data, 500 months is roughly 42 years. We observed in our experiments however, that things started to go awry much sooner than 500 observations, closer to 175, which with monthly data would correspond to just under 15 years. It is important to emphasize that working with shorter time series does not mean that we might not encounter the sort of estimation issues that we have been discussing here. Saddle-point dynamics, and trajectories like F_1 and F_2 are predicted outcomes of individual level intertemporal optimization problems. While we will in general not have a continuous time series of data on any individual through their entire life, we increasingly have available to us panel data sets, containing longitudinal data for individuals at various stages in the life course. Thus while we might not be able to follow a single individual along a trajectory such as F_1 through their entire life, we could reasonably hope to have longitudinal data on a series of individuals who, between them, spanned the whole of F_1. Thus there is some interest in the question of how the effects we are looking at here translate to panel data sets.

In actual panel data sets, estimation will be complicated by the fact that different age groups of individuals in the panel in any year might, as a result of different histories up to that year, have different values of their explanatory variables. For our purposes, the possibility that different individuals might display similar intrinsic dynamics (i.e. the same coefficients on the lagged dependent variable across individuals so that all individuals had the same true values of their characteristic roots) but along shifted trajectories adds a layer of complication with which we prefer not to deal here. In addition, because we are assuming that all individuals follow the same trajectory through their lives, we are assuming away the identification problem that underlies the Dynamic Panel Data econometrics literature (see Arellano (2003)). We propose to leave those issues for future research.

Thus, if we consider trajectory F_1 in Figure 1, in what follows we are essentially assuming that everyone follows exactly the same trajectory, so that when we put together a panel of individuals of different ages, our various individuals are at different points along exactly the same trajectory. This makes it quite possible that, so long as our sample of individuals covers the complete life span, we have a full set of observations all along the trajectory. Clearly, we are, for the moment, assuming away birth cohort effects as well as individual differences in the values of the exogenous variables that determine the location of the equilibrium. That type of identification issue we assume away, in order to focus on the way the effect that we have seen in pure time series data might manifest itself in panel data.

For purposes of our next Monte Carlo experiments, we assume that we have a data set consisting of 50 individuals, for each of whom we have ten observations, meaning that we are using our entire data set. We assume that none of the individual panels overlap with any others. We use the same data sets as we did for the pure time series runs, and use a recursive regression approach in which we add individuals rather than observations, so each iteration of our recursive estimation process adds ten observations from one individual's data set.

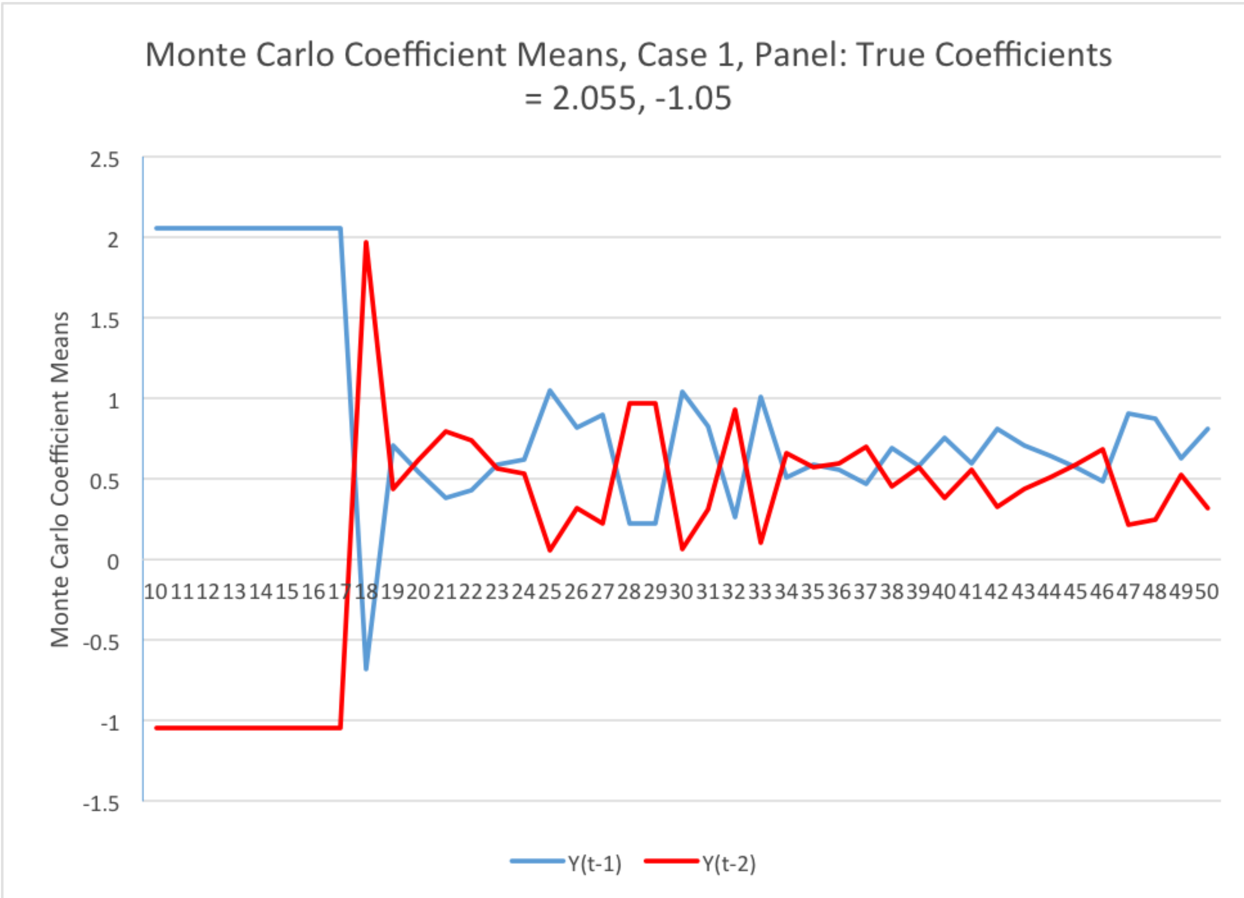

Figure 8: Monte Carlo Means, Panel Version of Case 1

The general pattern of the recursive panel coefficients in Figure 8 is very similar to that of the pure time series recursive coefficients in Figure 2, hardly surprising, since we used the same data and entered it in the recursive regressions in the same order, the only difference being that in the case of Figure 8 we added the data in blocks, not as individual observations. It is important to remember that we are running our experiments using recursive regression, and that in an actual panel data regression exercise we would only observe one pair of coefficient values. If, for example, we used all fifty of our individuals we would find the coefficient on the first lag to be 0.81 and that on the second lag, 0.32. While these are nowhere near the true values from the data generating process, 2.055 and -1.05 respectively, it is not clear that they would sound any warnings, although the fact that one of the roots was negative, a highly unusual occurrence in theoretical dynamic economic models might, were the roots to be calculated.

The panel counterpart of Case 2 is shown in Figure 9 below:

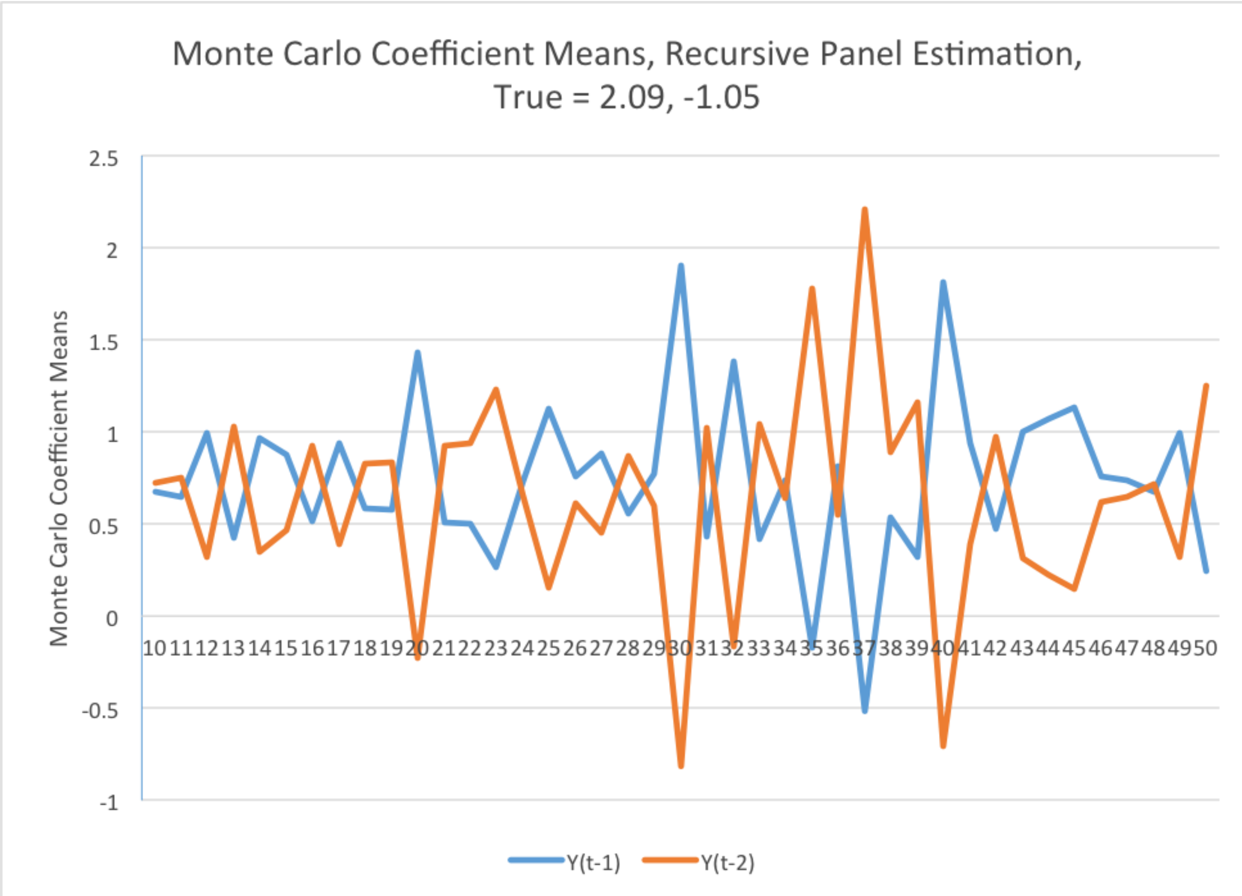

Figure 9: Monte Carlo Panel Estimation, Case 2

Here, because of the short well-behaved range in Figure 4 above, and the need to reserve some observations to initialize our recursive estimation, none of the means of our panel coefficient estimates come close to the true coefficient values from the DGP.

The panel data sets that we are using here are highly artificial, consisting of 50 individuals, with ten observations on each, effectively meaning that we have one individual from each age range. In practice, we would have different numbers of individuals from various different age ranges, or, equivalently, from various different intervals, along F_1 in Figure 1 (or a qualitatively similar trajectory depending on the model which we are estimating). In that case we would expect the estimated coefficients on the two lags to be weighted averages of the coefficients for each interval, with the weight depending on the proportion of the sample taken from the well- behaved and the badly-behaved ranges of the true trajectory: i.e. from regions where the stable root dominates and regions where the unstable root dominates (as well as the region where they would have similar weight, around the turning point on the trajectory). Whether the estimated coefficients would represent the true DGP at all well would depend on the age mix of the panel data set.

V Conclusions

We have chosen our coefficients from a relatively narrow range, with, for the most part, roots not far from 1. We did this because it seems unlikely that in economic applications we would find the unstable root dominating from the beginning. As we saw in Case 3 above, the relative sizes of the stable and unstable roots can be expected to be a determining factor in the size of the well-behaved region. We would also expect that where the true trajectory lies in Figure 1 - whether closer to the stable and unstable branches or farther away - would be a key issue. Regardless of the roots, we would expect a finite horizon trajectory that spent a longer part of its horizon close to the stable branch to have a larger well-behaved region. Assuming that most dynamic economic models do not have trajectories that are dominated by the unstable root early on, there remains the issue of whether the unstable root is sufficiently powerful in any particular application that the estimated coefficient values, while wrong, would not look entirely implausible.

In the Monte Carlo results presented above, we are not estimating the roots directly but rather are estimating difference equation coefficients, non-linear combinations of which yield the roots. As we structured the data sets, in the first part of each both the stable and unstable roots are effective in influencing the trajectory of the dependent variable, with the stable root dominating so long as the trajectory is heading in the direction of the equilibrium point while the unstable root comes to dominate once the trajectory swings away from the equilibrium point. In the region of the data set in which both roots influence the trajectory, even when the stable root is the dominant one, both the stable and unstable roots are reliably estimated. Once we move into the region where the unstable root dominates, however, the stable root becomes very poorly estimated as its influence on the trajectory of the dependent variable wanes. In the convergent region in which both roots are reliably estimated, the coefficients on the two lags of the dependent variable are also well estimated, but once we move clearly into the divergent region neither lag coefficient is well estimated.

One interpretation of this result might be that, in the region in which both roots are effective in determining the dynamics of the dependent variable, there is enough information present for the software to be able to fit the coefficients which make up the roots, but once we move into the region in which the stable root is observationally ineffective, the system is in a sense under-determined in so far as finding values of the lag coefficients is concerned. Any set of coefficients which will yield the unstable root will do, and, in the divergent region, the stable root does not exert enough influence to allow the software to narrow the choice down to that pair of values which also yield the stable root. The considerable variability which we observe in our Monte Carlo estimates of the coefficients suggests that, subject to the condition that their values yield the unstable root, the actual magnitudes of the estimated coefficients may be heavily influenced by the disturbance term. The graphs also suggest a tendency for the estimated coefficients to converge on a relatively narrow range of values, conditional on their values being consistent with the unstable root. This may reflect the presence of a high degree of multicollinearity in the data as the sample moves out along the trajectory, which may in turn be resulting in the program assigning very similar importance to the two lags. This is, however, still a conjecture.

One unexpected result of our experiments, which is at least on the surface consistent with this interpretation, was that the way the unstable root manifests itself seems to depend on the software being used[3]. The experiments reported above were programmed in R. We also ran similar pure time series recursive regression experiments using the PcNaive Monte Carlo package in PcGive (Doornik and Hendry (2013a,b)). In the PcNaive runs, while the well-behaved region coincided closely with that from our R runs, the badly-behaved region manifested quite differently. In our PcNaive experiments, while in the well-behaved region we obtained accurate estimates of the coefficients on both the first and second lags, once in the badly-behaved region the software simply set the coefficient on the second lag to zero and set the coefficient on the first lag to the value of the unstable root: in other words, in the well behaved region our recursive estimation results yielded a second order difference equation with saddle-point dynamics while once observations from the badly-behaved region were added to the data the program settled on an unstable first order difference equation. Thus whether we observed any warning signs related to the type of behavior which we have been considering here might well depend on the software we happen to be using, an issue which was once regarded as significant in econometrics but which in more recent years has implicitly been assumed to have been taken care of by increasing computer power. While observing results along the lines which our PcNaive runs gave us might indicate problems, if we found results similar to the ones reported here, generated from our experiments using R, we might accept them as plausible and possibly take them as evidence against the model which we were testing, if our theoretical work had given us some idea of possible coefficient values. If the theoretical work did not yield such values, we might take the sort of values reported here at face value.

In the case of panel data sets, one possible indicator of whether there is an issue of the sort which we have been discussing might involve sorting the data in order of increasing age and running the dynamic model block-recursively, to see whether instability seems to manifest itself in the estimated coefficients.

It is probably fair to say that most dynamic econometric modeling, including the DPD methodology focuses on first order difference equations. This is in part because not many economic models are easily seen to yield second order difference equations as solutions. Many dynamic economic models yield saddle-point phase diagrams, but we have not often in the past had the sort of individual level panel data sets which might permit us to test the dynamic predictions of the theoretical models as rigorously as we might have liked, and the type of second order equation which we have been discussing here is derived most commonly as a reduction of a system of two interrelated first order difference equations. As we have already noted, macro-dynamic models often also yield saddle-point phase diagrams, but since along the stable branch the stable root is driving the trajectory of the variable being observed, there should be no issues of the sort considered here.

While second order saddle-point dynamic equations are not as common as first order difference equations, however, the preliminary results reported here suggest that there may be interesting econometric issues associated with estimating such cases as do arise.

[3]Finding differences in the results of calculations depending on the software being used was once not that uncommon, but in recent years advances in the software have made it less of an issue. In our case, it turns out to be relevant. See the related observation by David Giles in a post on his blog Econometrics Beat "Statistical Calculations and Numerical Accuracy", posted June 6, 2015, at `http://davegiles.blogspot.ca/2015/06/statistical-calculations-numerical.html`, accessed June 9, 2015.

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