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The Use of Panel Quantile Regression for Efficiency Measurement: Insights from Monte Carlo Simulations

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Abstract

In panel stochastic frontier models, the Fixed Effects (FE) approach produces biased technical efficiency scores when time-invariant variables are important in the production process, and the Random Effects (RE) approach imposes distributional assumptions about the inefficiency. Moreover, technical efficiency scores obtained from these models are biased when the sample contains a large number of firms near the efficient frontier. We propose the use of quantile regression (QR) with a Correlated Random Effects (CRE) specification as an alternative to these approaches. Using Monte Carlo simulations, we show that CRE QR can overcome the limitations of FE and RE stochastic frontier models.

JEL Classification: C23; D2

Keywords: technical efficiency; quantile regression; panel data; stochastic frontier analysis

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1 Introduction

In a panel context, efficiency[2] analysis is usually performed using stochastic frontier models, which are estimated using fixed effects (FE) and random effects (RE) (Greene, 2005). The FE approach is generally seen as attractive as it does not make distributional assumptions on the inefficiency and allows the explanatory variables to be correlated with the inefficiency term (Jacobs et al, 2006). However, the FE estimator does not allow for the inclusion of time-invariant variables in the analysis, which implies that all between-firm characteristics (observed and unobserved) goes into the same term that is used to calculate inefficiency (Greene, 2005). This can lead to biased technical efficiency scores if these firm-level characteristics significantly affect the production process (Feng and Horrace, 2007). The RE model allows for the inclusion of time-invariant variables but imposes distributional assumptions with regard to the inefficiency and assumes independence of the explanatory variables and the inefficiency term (Jacobs et al, 2006). Both methods assume one firm is 100% efficient, with the other firms ranked in relation to this firm (Kumbhakar and Lovell 2000). This ranking mechanism has been shown to produce biased estimates of the technical efficiency scores when the sample consists of a large number of efficient firms (Feng and Horrace, 2012).

A possible alternative to panel stochastic frontier models is the semi-parametric Quantile regression (QR) approach. Using Monte Carlo simulations, Liu et al. (2008) showed that QR outperformed the Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA)[3] approaches in the calculation of technical efficiency scores when the sample consisted of a large number of efficient firms. This analysis was performed using simulated cross-sectional data.

The use of QR for panel data has been complicated by the issue of "unobserved heterogeneity", or unobserved firm-level characteristics that are not adequately controlled for by the model. In the context of efficiency analysis, this would represent inefficiency (assuming all other factors in the production process are controlled for in the model). Although QR has been used for efficiency analysis using panel data (see Knox et al (2007) and Koutsomanoli-Filippaki et al. (2013)), these studies do not address potential correlation of the explanatory variables and the inefficiency term

This paper discusses the use of panel QR as an alternative to panel stochastic frontier models. The estimator we propose, which is based on the "correlated random effects" (CRE) specification presented in Bache, Dahl, and Kristensen (2013), does not make assumptions about the distribution of inefficiency, allows for the inclusion of time-invariant variables, and produces unbiased estimates of the slope parameters when the time-varying variables are correlated with the inefficiency term. Using Monte Carlo simulations, we show that this specification for panel QR at the 80th, 85th, and 90th percentiles outperforms the FE and RE estimators, regardless of the number of efficient firms in the sample. To the best of our knowledge, our study is the first to compare the performance of SFA (FE & RE) and QR in a panel context using simulated data. The results of the simulations suggest that panel CRE QR may be used as an alternative to stochastic frontier models in applied work where the true distribution of inefficiency is never known.

This paper is organized as follows. Section 2 presents a review of stochastic frontier models for

[2]In this paper, we deal with the concept of technical efficiency. A firm is said to be technically efficient if conditional on its inputs, its output mix lies on the production possibility frontier (Liu et al, 2008).

[3]DEA is a non-parametric approach to calculating technical efficiency scores (Jacobs et al, 2006). We do not explore this estimator in this paper.

panel data. Section 3 provides an overview of the use of QR for efficiency analysis using panel data. Section 4 describes the Monte Carlo methods used in this study. Section 5 presents the results of the Monte Carlo simulations. Section 6 applies SFA and CRE QR to a real-world dataset consisting of Indonesian Rice Farmers. Section 7 concludes.

2 Stochastic Frontier Analysis Using Panel Data

The Stochastic Frontier Model for panel data can be written as

$$y_{it} = \alpha + x_{it}^{'}\beta + z_{i}^{'}\gamma - \mu_i + v_{it}, \qquad i = 1, ..., N, t = 1, ..., T \tag{1}$$

where $x_{it}^{'}$ is a vector of time-varying variables, $z_i^{'}$ is a vector of time-invariant variables, $\mu \geq 0$ is the time-invariant, technical inefficiency term, and v_{it} is the usual stochastic error term, distributed i.i.d $(0, \sigma_{v_{it}^2})$.

Broadly speaking, there are two main goals in estimating SFA models such as (1): i) consistent estimates of the slope parameters (i.e. β and γ) and ii) consistent estimates of the technical efficiency term, μ_i.

2.1 Estimation of the slope parameters

FE estimation of (1) can be achieved by including individual intercepts for each firm i in the dataset, or through the "within" deviation from individual mean transformation (Kumbhakar and Lovell 2000). In either case, time-invariant variables cannot be included in the estimation (and hence estimates of γ cannot be obtained). Despite this, the FE estimates of the time-varying variables (β_{FE}) are generally thought to be unbiased estimates of the true population parameters (β) since they control for all time-invariant factors in the analysis (observed and unobserved).

The alternative to the FE estimator is the RE estimator, which can be estimated via Feasible Generalized Least Squares (GLS) or Maximum Likelihood (Jacobs et al, 2006). Unlike the FE estimator, which only uses variation within firms for identification, the RE estimator makes use of both within and between firm variation (Greene, 2012). Hence, unlike the FE approach, RE estimation allows for the inclusion of time-invariant variables, and so one is able to obtain estimates of the effects of these variables on output (i.e γ_{RE}). However, the RE estimator assumes that the explanatory variables are uncorrelated with the time-invariant, inefficiency term. Violation of this assumption leads to biased estimates of the slope parameters (β_{RE} & γ_{RE}).

Mundlak (1978) proposed a modification of the RE framework that allows for correlation of the time-varying variables and the μ_i term. In this sense, the unobserved individual characters can be seen as a projection of the time-means of the time-varying variables plus a disturbance term, or:

$$\mu_i = \theta \bar{x}_i + \delta_i \tag{2}$$

where $\bar{x_i} = \frac{1}{T}\sum_{t=1}^{T} x_{it}$ and $\delta \sim N(0, \sigma_\delta^2)$.

Using Mundlak's device, it can be shown that the RE estimates of the time-varying variables will be the same as the FE estimates, or $\beta_{RE} = \beta_{FE}$ (see, for example, Hsiao (2003)). Farsi (2005) proposes the use of Mundlak's device in the context of efficiency analysis.

To summarize, both the FE and RE (with Mundlak) specifications of the SFA model can be used to consistently estimate the parameters on the time-varying variables (β) in the analysis. Estimates of γ can only be obtained through the RE framework, although these estimates may be inconsistent if the time-invariant variables are correlated with the inefficiency term[4].

2.2 Estimation of the technical efficiency scores

Stochastic Frontier models characterize inefficiency as a one-sided probability distribution (Jacobs et al, 2006). In a cross-sectional setting, stochastic frontier models can only be estimated via maximum likelihood, and a distribution for the inefficiency needs to be specified (Jacobs et al, 2006). The choices available in most software packages are the half-normal, truncated normal, exponential, and gamma distributions (Jacobs et al, 2006). These specifications are all unbiased but inconsistent estimators of the inefficiency since the variance of the estimate remains non-zero, regardless of the sample size (Greene 1993, Jacobs et al, 2006). Moreover, there is no economic basis to guide the choice of distribution in applied work (Jacobs et al., 2006), and the results of the efficiency analysis are sensitive to the distribution chosen (Liu, Laporte, & Ferguson, 2008). Using panel data, it is possible to relax some of these assumptions.

Using a FE approach, technical inefficiency can be computed using the intercepts via (β_{MAX} - β), which are then put into the following form[5]:

$$te = exp(-\mu_i) \tag{3}$$

One of the main selling points of the FE estimator is that it does not impose a particular distribution for the inefficiency term (Jacobs et al, 2006). However, the effects of all time-invariant factors (observed and unobserved) go to the same term used to calculate the inefficiency. Feng & Horrace (2007) show that the estimates of the technical efficiency scores using the FE estimator are biased when time-invariant variables are important inputs in the production process.

Under a RE framework using Feasible GLS, the μ_i's can be calculated from the residuals, or:

$$\frac{1}{T}\sum_{t=1}^{T}(y_{it} - \alpha - x_{it}^{'}\beta - z_i^{'}\gamma) \tag{4}$$

[4]This would likely need to be solved using an Instrumental Variables (IV) approach (Rabe-Hesketh & Skrondal (2012)). This issue is not explored in this paper.

[5]See Kumbhakar and Lovell (2000) for a detailed description.

After the normalization (μ_{MAX} - μ), the μ_i's can be put into equation (3) (Kumbhakar and Lovell, 2000). Unlike the FE estimator, the RE approach allows us to directly control for time-invariant variables, thereby allowing us to distinguish between their effects and inefficiency. However, the RE GLS approach requires mild distributional assumptions about the inefficiency - specifically, that the μ_i are randomly distributed with constant mean and variance (Kumbhakar and Lovell 2000).

The RE approach can also be estimated using Maximum Likelihood (ML), but this requires stronger assumptions about the shape of the distribution than GLS, with choices generally stemming from the cross-sectional literature (Jacobs et al, 2006). Moreover, ML estimation may be sensitive to the iterative and updating processes, so the results may reflect convergence to a local rather than global maximum (Jacobs et al., 2006). In this paper, we limit our discussion to RE estimated via GLS.

Thus, if the only source of between-firm variation is technical inefficiency after all inputs are controlled for, we would expect to see similar technical efficiency scores between the FE and RE approaches. In this case, the FE approach may be preferred[6] since one does not need to make distributional assumptions about the inefficiency. If there are other observable time-invariant factors that affect the production process (e.g. profit status, rural location, chain affiliation, etc.), then the RE approach may be preferred , since observable firm characteristics can be directly controlled for, at the price of making distributional assumptions about the inefficiency term.

In terms of the calculation of the technical efficiency scores, both methods assume one firm is 100% efficient, with the remaining firms ranked in relation to this firm (Kumbhakar and Lovell 2000). As noted above, Feng and Horrace (2012) show that the estimates of technical efficiency obtained from the FE estimator are biased when the distribution of efficiency is skewed (i.e. when there is a large number of highly efficient firms). They go on to note that this is likely the case in competitive markets where inefficient firms are rare. In their paper, they propose forming the frontier relative to the worst performing firm in the data (as opposed to the best) using a FE approach. However, in their empirical application, they ignore the bias of the FE estimator when time-invariant variables are important in the production process. They also don't explore the implications of the bias in a RE framework.

3 Quantile Regression for Estimation of Slope Parameters and Technical Efficiency

QR is a semi-parametric approach, which models conditional quantiles as a function of predictors (Hao & Naiman, 2007). This is in contrast to traditional estimators, including stochastic frontier models, which model the conditional mean. In this sense, QR is less restrictive than conditional mean models as it allows for the possibility that the effect of covariates varies across the quantiles.

QR does not make distributional assumptions about the error term. Similar to the linear model, it requires that the mean of the disturbances is equal to 0. The number of observations with a positive or negative residual depends on the quantile estimated. For example, at the 90th percentile, 10% of

[6] Feng and Horrance (2007) propose an alternative way to calculate technical efficiency in a FE setting with time-invariant variables in the analysis. We do not explore this here.

firms are given a positive residual, and 90% of firms are given a negative residual. Hence, in terms of technical efficiency scores, we would expect QR at the 90th percentile to perform well when the sample consists of fewer efficient firms.

Liu et al (2008) performed Monte Carlo simulations to investigate the performance of QR in comparison to SFA and DEA in a cross-sectional context. They found that QR at the 80th percentile outperformed SFA and DEA when the sample consisted of a large number of fully-efficiency firms. QR also did well with fewer efficient firms, but tended to over-estimate the efficiency of the more efficient firms in the sample. This may suggest that QR can be used as a viable alternative to SFA FE and RE in a panel context when the sample consists of a large number of efficient firms.

The most natural extension of QR to panel data would be to simply pool the observations together. Once this is done, one could extend the Liu et al. (2008) definition of inefficiency in a quantile context as the distance between a firm's observed and actual levels of output, apply this to each year, and then average across time (see equation 4), which can then be inserted into equation (3). This raises several concerns. First, the coefficients on the covariates may be biased if correlated with the inefficiency term μ_i. The other issue is the dependence of observations over-time, implying that a conventional standard error for QR may be inappropriate.

This first issue can be solved for the time-varying variables with a "correlated random effects" type framework for QR. This was first applied to QR by Abrevaya and Dahl (2008), which was an extension of the Chamberlain (1982) approach. Bache et al (2013) further extended the approach to unbalanced panels using the Mundlak (1978) specification. Although called "correlated random effects", the approaches do not fully follow the linear version, as no actual random effects are added to the model. Instead, a blocked pairwise subsampling bootstrap is used to account for the dependence of observations over time.

In addition to the above, there have been other approaches to panel QR that are closer to a SFA type approach. Koenker (2004), for example, developed a FE type procedure for QR, but this approach imposes a penalty factor to shrink the individual effects toward a common intercept. Although it is possible to set this penalty factor to zero to obtain estimates of the FE, one would need to restrict their effect to be the same across the quantiles.

RE estimation has also been introduced for panel QR as an alternative to the FE procedure. As its name suggests, the RE approach involves the estimation of random intercepts for each individual in a given sample (Geraci and Bottai, 2007; Liu and Bottai, 2009). While this approach takes into account the dependence of observations over time, independence of the random intercepts and the explanatory variables is assumed. When these assumptions are violated, estimates produced using the RE model may be biased. Moreover, one would need to make distributional assumptions on the RE term.

For this paper, we restrict our attention to the Bache et al (2013) specification since it has the desirable attribute of not making distributional assumptions on the inefficiency and allows for correlation between the explanatory variables and the inefficiency term.

4 Methods

In this paper, we compare the performance of SFA-FE, SFA-RE, and QR at the 80th, 85th, and 90th percentiles in the estimation of technical efficiency using Monte Carlo simulations. We generate output from a Cobb-Douglas production function with multiple inputs, which takes the form:

$$y_{it} = \beta_0 x_{1it}^{\beta_1} x_{2it}^{\beta_2} x_{3it}^{\beta_3} x_{4it}^{\beta_4} e^{\gamma z_i} e^{-\mu_i} e^{v_{it}} \tag{5}$$

where i=1,...,n, n is the number of firms, t=1,...t, t is the number of time periods.

The technical efficiency term μ_i is initially generated from the half-normal distribution (exponential later on). The error term $v_i t$ is generated from the normal distribution (v$_i t \sim$ i.i.d (0, 0.01)). Inputs are generated from the uniform distribution. We also generate a time-invariant dummy variable z_i, which in the context of efficiency measurement could represent a firm characteristic that significantly affects the production process (i.e. profit status, rural location, chain affiliation, etc.). Following Liu et al (2008) we choose values of the coefficients on the inputs so the production function exhibits decreasing returns to scale. Summary statistics for the starting values of the Monte Carlo data are presented in Table 1.

[Table 1 here]

In our simulations, the same technical efficiency term is used throughout each of the Monte Carlo replications. This allows us to see how the estimators perform in calculating the 'true' technical efficiency across the Monte Carlo replications.

As the FE estimator requires T $\rightarrow \infty$ for consistent estimates of the fixed effects (Greene, 2012), we chose T=5 with n=100 to avoid the "incidental parameters" problem. Each experiment is run using 500 replications.

All analysis was performed using R version 3.2.2 "Fire Safety" (R Core Team, 2013). The CRE QR specifications were estimated using the 'rqpd' package (Koenker and Bache, 2011). The SFA FE & RE specifications were estimated using the 'plm' package (Croissant & Millo, 2015).

5 Monte Carlo Results

5.1 Consistency of estimators for slope parameters and technical efficiency scores

In our first set of results, we work under conditions that are most favorable for a panel SFA type analysis. More specifically, we set $\gamma = 0$ so that the only source of between firm variation (other than the inputs) comes from the technical inefficiency term. We also specify 1 firm to be fully efficient ($\mu_i = 0$), and do not allow for correlation between the explanatory variables and the inefficiency term. As a result, we do not use the Mundlak specification for SFA-RE or QR.

Table 2 presents the estimates of the coefficients across the 500 Monte Carlo replications. We can see from Table 2 that all 5 estimators produce unbiased estimates of the coefficients.

Figure 1 presents the average technical efficiency scores for each firm across the 500 replications. SFA-FE and SFA-RE perform nearly identically - generally underestimating the technical efficiency scores of all firms in the sample. QR at the 90th percentile generally underestimates the efficiency of all firms in the sample, but produces a frontier that is closer to the true than SFA (FE and RE). QR at the 85th and 90th percentiles generally overestimate the efficiency of all firms in the sample, but QR at the 85th comes quite close to the true. In general, the technical efficiency scores at the 85th and 90th percentiles appear to come closest to the true.

[Table 2 here]

[Figure 1 here]

In our second experiment, we impose correlation between the explanatory variables and the inefficiency term. In this scenario, we do not use the Mundlak specification. This allows us to explore the consequences of ignoring this correlation on the beta coefficients and technical efficiency scores.

Table 3 presents the coefficient estimates in this scenario. The FE estimator produces unbiased estimates of all slope coefficients, which is consistent with our expectations. The RE estimates are biased, but to a very small degree. We see the most amount of bias in the betas from the pooled QR.

[Table 3 here]

Figure 2 presents the results of the technical efficiency scores from this experiment. From this, we can see that the RE estimator still produces estimates of the inefficiency that are on par with the FE approach. QR performs about the same as the previous experiment.

[Figure 2 here]

In our third experiment, we apply the Mundlak specification to SFA-RE, as well as QR at all percentiles. The results of the beta coefficients are shown in Table 4. By applying the Mundlak specification, we are able to produce approximately unbiased estimates of the coefficients in the SFA-RE and Quantile models.

[Table 4 here]

In terms of the technical efficiency scores, the Mundlak specification appears to help the SFA-GLS specification in identifying the least efficient firms, but produces downward bias on the technical efficiency scores of the more efficient firms (see Figure 3). The technical efficiency scores of QR specifications are relatively unchanged.

[Figure 3 here]

These results seem to suggest that QR at the 85th and 90th percentiles outperform SFA (FE and RE) under conditions that SFA should strive in (i.e. 1 firm fully efficient, technical efficiency generated from a distribution with constant mean and variance, no time-invariant inputs in the production process). The technical efficiency scores of QR are largely unaffected by the correlation of the inputs and the inefficiency, but the beta coefficients are biased. This, however, is corrected for by Bache et al.'s (2013) specification.

5.2 Technical efficiency scores with more than one fully efficient firm

In our second set of experiments, we begin to move away from the unrealistic scenario where only one firm is fully efficient in the sample. In all experiments hereafter, we impose correlation between the inefficiency term and the inputs (to more accurately reflect market conditions) so the SFA-RE and QR specifications are estimated with the Mundlak specification. Since all 5 estimators allow for correlation between the time-varying variables and the inefficiency term, we do not report coefficients for the next sets of results (since they are all unbiased).

In our first set of results, we specify 15 firms to be fully efficient in the sample. The results are shown in Figure 4. We can see from this figure that both SFA-FE and SFA-RE underestimate the technical efficiency scores of all firms in the sample. QR at all percentiles accurately identifies the fully efficient firms in the sample, but QR at the 85th and 90th do a better job of ranking the less efficient firms than QR at the 80th percentile.

In our next experiment, we increase the total number of fully efficient firms to thirty-four. The average technical efficiency scores are presented in Figure 5. In this set of results, it appears QR at the 85th percentile performs the best. QR at all percentiles seems to do well in identifying the efficient and inefficient firms, but QR at the 85th percentile generally produces technical efficiency scores that are closest to the true.

[Figure 4 here]

[Figure 5 here]

In our next experiment, we specify forty-nine firms to be fully efficient in the sample (Figure 6). In this specification, QR generally performs well at all three percentiles. SFA-FE performs almost as well as QR at the 90th percentile. QR at the 80th percentile outperforms QR at the 85th and 90th percentiles, which makes intuitive sense since the sample consists of a large number of efficient firms (>20%).

[Figure 6 here]

In our last experiment for this section, we impose sixty-four firms to be fully efficient in the sample. The results are presented in Figure 7. The results are consistent with expectations - since the sample consists of a large number of efficient firms, quantile at the 80th percentile outperforms the other estimators. SFA-FE and QR at the 90th percentile perform almost identically, but QR at the 90th percentile does a better job of ranking the less efficient firms.

Table 5 presents the mean technical efficiency scores of the 4 experiments in this section. We can see from this table that SFA-RE consistently underestimates the mean technical efficiency score, regardless of the number of firms in the sample. SFA-FE does quite a bit better - producing higher mean technical efficiency scores as the number of efficient firms increases. QR at any given percentile tends to produce higher technical efficiency scores than SFA-RE and SFA-FE. Although the mean technical efficiency scores tend to be underestimated with a large number of efficient firms, this is likely due to our definition of technical efficiency (i.e. those on or above the frontier are considered to be 100% efficient). We can see from Figure 7 that QR at all percentiles generally produces technical efficiency scores ≥ 0.9 for all fully efficient firms, and scores that are close to the true for the more inefficient firms. Thus, in terms of identifying the efficient/inefficient firms, QR seems to do quite well at all percentiles, although SFA-FE is a strong competitor.

[Figure 7 here]

[Table 5 here]

5.3 Technical efficiency scores with time-invariant variables

In our next set of experiments, we set $\gamma = 1$ (coefficient on the time-invariant variable dummy variable, z_i) to introduce an additional source of between firm variation in the production process. We are able to control for this variable directly using SFA-RE and QR, but are forced to omit it in the SFA-FE specification (since it is time-invariant).

Figure 8 presents the technical efficiency scores for this scenario with only one firm fully efficient. We can see from this figure that the technical efficiency scores of SFA-FE are all biased downwards. SFA-RE performs much better than SFA-FE (since we are able to control for zi directly) but QR at the 90th percentile seems to perform the best. QR at the 80th percentile overestimates the efficiency of all firms - but this is to be expected given only firm is fully efficient (again, a situation that is unlikely to occur in real world data).

The bias of SFA-FE persists as we increase the number of efficient firms from one to thirty-four (see Figure 9). Performance of QR at 80th, 85th and 90th are roughly the same from the previous case where the only firm level characteristic was inefficiency. Technical efficiency scores from SFA-RE are biased downwards for all firms in the sample.

Table 6 presents the mean technical efficiency scores for this section. We can see that both SFA-FE and SFA-RE underestimate the true average mean, with SFA-RE producing scores that are closer to the true. QR at all percentiles produces estimates that are generally quite close to the true.

[Figure 8 here]

[Figure 9 here]

[Table 6 here]

5.4 Technical efficiency scores with exponential inefficiency

In our next set of experiments, we compare the estimators using a different distribution for the inefficiency term - namely the exponential. This distribution clusters most observations near the efficient frontier with a long tail of observations further away (Jacobs et al., 2006). Hence, we may expect this type of distribution to appear in real-world applications where the market consists of a large number of efficient firms.

In the next case, we set $\gamma = 0$ (to avoid the bias of the FE estimator) and set 1 firm to be fully efficient. The results of the simulations are presented in Figure 10. Interestingly, QR at the 80th and 85th percentiles seem to outperform SFA-FE and SFA-RE for every firm in the sample. Estimates produced by QR at the 80th percentile tend to overestimate the efficiency of all firms - but do so to a greater extent for highly efficient firms and tend to do a good job of estimating the inefficiency of inefficient firms in the sample.

[Figure 10 here]

Increasing the number of fully efficient firms from one to thirty-four (see Figure 11) seems to help the SFA-FE and QR estimators, but QR at all percentiles seems to dominate the SFA approaches.

Table 7 presents the mean technical efficiency scores of the estimators. QR at all percentiles generally produces mean technical efficiency scores that are close to the true, although SFA-FE is close as well (since there are no time-invariant variables in the production process).

[Figure 11]

[Table 7 here]

6 Empirical Application

We now apply SFA-FE, SFA-RE, and CRE QR at the 80th, 85th, and 90th percentiles to the Indonesian Rice Farmers dataset, which is commonly used in the SFA literature (Feng and Horrace, 2012), starting with Erwidodo (1990). The data consists of 171 farmers from 6 different villages observed for 6 time periods. Output is the log of kilograms of rice produced. Inputs include seed (kg), urea (kg), trisodium phosphate (kg), labour (hours) and land (hectares) (Horrace & Schmidt 1996) (all expressed as natural logs). Time-varying dummy variables include pesticide use (DP), indicators for the type of rice varieties that are planted (DV1 for high yield varieties, DV2 for mixed varieties, with traditional varieties as the reference category), and an indicator DSS for the wet seasons (which take place in periods 1, 3, and 5) (Druska & Horrace, 2004). Time-invariant dummy variables are the 6 village indicators of the farms. A Cobb-Douglas production function is used for the analysis. All inputs consisted of values greater than zero except for trisodium phosphate, which contained zero values for some farmers. These zero values were recoded as 1 before the log transformation was applied.

Table 8 presents the results of this analysis for the various estimators examined in this paper. The SFA-RE and CRE QR specifications were run using the Mundlak specification. It should be noted that a Mundlak mean was not included for dss, since the mean of this variable was constant for all farmers. The SFA-FE and SFA-RE specifications produce identical estimates on the coefficients of the time-varying variables, but we are able to include the time-invariant region dummies in the SFA-RE specification. The mean technical efficiency score for SFA-FE is likely biased downwards without these indicators, which appears to be the case, since mean technical efficiency is higher in the SFA-RE specification.

[Table 8 here]

Similar to the SFA-RE specification, QR allows us to include the time-invariant region dummies in the regression. In terms of the inputs, there appears to be a non-uniform effect across output distribution, as the estimated elasticities appear to vary across the quantiles.

In terms of the technical efficiency scores, the mean efficiency at the 80th, 85th, and 90th percentiles, consistent with our simulation results are all higher than the SFA-FE and SFA-RE estimates.

7 Conclusions

In this paper, we investigate the performance of QR for the estimation of slope parameters and technical efficiency scores using panel data. We show that QR with a Mundlak (1978) specification (suggested by Bache et al (2013)) produces unbiased estimates of the slope parameters when the explanatory variables are correlated with the inefficiency term. We also show that QR may be preferred to traditional stochastic frontier approaches since it does not impose distributional assumptions with regard to the inefficiency term, allows for the inclusion of time-invariant variables, and performs well regardless of the number of efficient firms.

QR may therefore be another tool in the applied analyst's toolbox for assessing the efficiency of firms in a particular sector. If it is believed that the market consists of a large number of efficient firms, then QR at the 80th percentile may be preferred. That being said, as the true distribution of efficient firms is never known, it may be advisable to estimate technical efficiency scores based on frontiers estimated at the 80th, 85th and 90th percentiles in applied work.

References

Abrevaya J, Dahl CM. 2008. The effects of birth inputs on birthweight: Evidence from quantile estimation on panel data. *Journal of Business & Economic Statistics* 26: 379-397.

Bache SHM, Dahl CM, Kristensen JT. 2013. Headlights on tobacco road to low birthweight outcomes. *Empirical Economics* 44: 1593-1633.

Chamberlain G. 1984. *Panel data.* In: Griliches Z, Intriligator MD (Eds), Handbook of Econometrics, Vol 2. Elsevier Science B.V.: New York, NY.

Croissant, Y. and Millo, G. 2015. plm, Linear Models for Panel Data. R package version 1.5-12

Druska, V. and Horrace, W.C., 2004. Generalized moments estimation for spatial panel data: Indonesian rice farming. *American Journal of Agricultural Economics*, 86: 185-198.

Erwidodo. 1990. *Panel data analysis on farm-level efficiency, input demand and output supply of rice farming in West Java, Indonesia.* PhD dissertation, Department of Agricultural Economics, Michigan State University.

Farsi M, Filippini M, Kuenzle M. 2005. Unobserved heterogeneity in stochastic cost frontier models: An application to Swiss nursing homes. *Applied Economics* 37:2127-2141.

Feng Q, Horrace WC. 2007. Fixed-effect estimation of technical efficiency with time-invariant dummies. *Economics Letters* 95: 247-252.

Feng Q, Horrace WC. 2012. Alternative technical efficiency measures: skew, bias and scale. *Journal of Applied Econometrics* 27: 253-268.

Geraci M, Bottai M. 2007. Quantile regression for longitudinal data using the asymmetric Laplace distribution. *Biostatistics* 8(1):140-154.

Greene, W. 1993. 'The econometric approach to efficiency analysis', in Fried, H. O., Lovell, C. A. K. and Schmidt, S. S. (eds.), *The Measurement of Productive Efficiency.* Oxford: Oxford University Press, pp. 68-119.

Greene, W., 2005. Fixed and random effects in stochastic frontier models. *Journal of productivity analysis* 23: 7-32.

Greene, W. H. 2012. *Econometric analysis* (7th ed.). Upper Saddle River, N.J: Prentice Hall.

Hao L, Naiman DQ. 2007. *Quantile regression.* Sage Publications: Thousand Oaks, CA.

Horrace, W.C. and Schmidt, P., 1996. Confidence statements for efficiency estimates from stochastic frontier models. *Journal of Productivity Analysis*, 7: 257-282.

Hsiao, C. (2003) *Analysis of Panel Data*, 2nd edn, Cambridge University Press.

Jacobs R, Smith PC, Street A. 2006. *Measuring efficiency in health care: Analytic techniques and health policy.* Cambridge University Press: New York.

Knox KJ, Blankmeyer EC, Stutzman JR. 2007. Technical efficiency in Texas nursing facilities: A stochastic production frontier approach. *Journal of Economics and Finance* 31:75-86.

Koenker R. 2004. Quantile regression for longitudinal data. *Journal of Multivariate Analysis* 91: 74-89.

Koenker R, Bache SH. 2011. rqpd: Regression quantiles for panel data. R package version 0.5/r9.

Koutsomanoli-Filippaki, A., Mamatzakis, E. and Pasiouras, F., 2013. A quantile regression approach to bank efficiency measurement. In *Efficiency and Productivity Growth: Modelling in the Financial Services Industry*

Kumbhakar SC, Lovell CK. 2000. *Stochastic frontier analysis.* Cambridge University Press: Cambridge.

Liu C, Laporte A, Ferguson BS. 2008. The quantile regression approach to efficiency measurement: Insights from Monte Carlo simulations. *Health Economics* 17: 1073-1087.

Liu Y, Bottai M. 2009. Mixed-effects models for conditional quantiles with longitudinal data. *International Journal of Biostatistics* 5. Retrieved from doi:10.2202/1557-4679.1186

Mundlak Y. 1978. On the pooling of time series and cross section data. *Econometrica* 46: 69-85.

R Core Team. 2013. R: *A language and environment for statistical computing.* R Foundation for Statistical Computing: Vienna, Austria.

Rabe-Hesketh, S. and Skrondal, A., 2012. *Multilevel and longitudinal modeling using Stata. Volume I: Continuous Responses.* STATA press.

Tables

Table 1: DGP Data Description

	Mean	Std. dev.	Min	Max
X_1	3.344	1.154	1.1686	5.5687
X_2	8.8001	3.7701	2.1443	15.2567
X_3	8.3092	4.1012	1.242	15.3405
X_4	433.9715	170.0607	145.53	745.5886
Z	0.58	0.4941	0	1
μ_i (half-normal)	0.3235	0.209	0	0.9376
μ_i (exponential)	0.1919	0.1880	0	1.0361
True values	$\beta_0 = 50$, $\beta_1 = 0.2$, $\beta_2 = 0.1$, $\beta_3 = 0.08$, $\beta_4 = 0.15$, $\beta_5 = 1$			

Table 2: Coefficient Estimates - One Firm Fully Efficient, Explanatory Variables Uncorrelated with Inefficiency Term

	SFA - GLS		SFA - FE		QR 80th		QR 85th		QR 90th	
Coefficient	β	SD β	β	SD β	β	SD β	β	SD β	β	SD β
β_0	3.655	0.071	NA	NA	3.873	0.143	3.894	0.142	3.922	0.157
β_1 (0.20)	0.2	0.011	0.2	0.011	0.2	0.021	0.201	0.022	0.201	0.023
β_2 (0.1)	0.1	0.01	0.1	0.01	0.099	0.018	0.1	0.018	0.1	0.019
β_3 (0.08)	0.08	0.008	0.08	0.008	0.08	0.015	0.08	0.015	0.08	0.016
β_4 (0.15)	0.15	0.011	0.15	0.011	0.149	0.022	0.15	0.022	0.151	0.024
Mean TE (0.793)	0.744		0.744		0.828		0.809		0.782	

Table 3: Coefficient Estimates - One Firm Fully Efficient, Explanatory Variables Correlated with Inefficiency Term

	SFA - GLS		SFA - FE		QR 80th		QR 85th		QR 90th	
Coefficient	β	SD β	β	SD β	β	SD β	β	SD β	β	SD β
β_0	3.665	0.072	NA	NA	3.943	0.145	3.956	0.148	3.978	0.16
β_1 (0.20)	0.196	0.012	0.2	0.012	0.166	0.021	0.169	0.023	0.174	0.024
β_2 (0.1)	0.099	0.01	0.1	0.01	0.091	0.018	0.092	0.018	0.093	0.019
β_3 (0.08)	0.079	0.008	0.08	0.008	0.073	0.015	0.073	0.016	0.075	0.016
β_4 (0.15)	0.15	0.011	0.15	0.011	0.148	0.022	0.15	0.023	0.15	0.025
Mean TE (0.793)	0.744		0.743		0.83		0.81		0.783	

Table 4: Coefficient Estimates - One Firm Fully Efficient, Explanatory Variables Correlated with Inefficiency Term, Mundlak Specification

	SFA - GLS		SFA - FE		QR 80th		QR 85th		QR 90th	
Coefficient	β	SD β	β	SD β	β	SD β	β	SD β	β	SD β
β_0	4.601	0.841	NA	NA	4.241	0.522	4.222	0.511	4.208	0.522
β_1 (0.20)	0.2	0.012	0.2	0.012	0.202	0.02	0.202	0.022	0.202	0.026
β_2 (0.1)	0.1	0.01	0.1	0.01	0.1	0.018	0.099	0.019	0.099	0.02
β_3 (0.08)	0.08	0.008	0.08	0.008	0.081	0.015	0.081	0.016	0.082	0.017
β_4 (0.15)	0.15	0.011	0.15	0.011	0.149	0.02	0.148	0.022	0.149	0.025
Mean TE (0.793)	0.701		0.743		0.837		0.815		0.787	

Table 5: Technical Efficiency Scores as Number of Fully Efficient Firms Increases

Number of Efficient Firms	TE (True)	SFA-RE	SFA-FE	QR 80th	QR 85th	QR 90th
15	0.796	0.694	0.736	0.834	0.813	0.784
34	0.81	0.683	0.738	0.829	0.803	0.773
49	0.83	0.68	0.75	0.821	0.797	0.77
64	0.859	0.686	0.774	0.824	0.804	0.78

Table 6: Technical Efficiency Scores, 1 Time-Invariant Variable in Production Process

Number of Efficient Firms	TE (True)	SFA-RE	SFA-FE	QR 80th	QR 85th	QR 90th
1	0.793	0.699	0.516	0.837	0.815	0.788
34	0.81	0.68	0.512	0.83	0.804	0.774

Table 7: Technical Efficiency Scores, Exponential Technical Efficiency

Number of Efficient Firms	TE (True)	SFA-RE	SFA-FE	QR 80th	QR 85th	QR 90th
1	0.834	0.75	0.781	0.861	0.839	0.812
34	0.847	0.735	0.772	0.854	0.83	0.802

Table 8: Indonesian Rice Farmers, Coefficient Estimates and Technical Efficiency Scores

Variable	SFA-FE	SFA-RE	QR - 80th	QR - 85th	QR - 90th
Intercept		5.228***	6.747***	6.565***	5.756***
		(0.435)	(0.513)	(0.566)	(0.629)
logseed	0.12***	0.12***	0.114**	0.142***	0.193***
	(0.03)	(0.03)	(0.049)	(0.053)	(0.064)
logurea	0.089***	0.089***	0.121***	0.117***	0.104**
	(0.021)	(0.021)	(0.042)	(0.043)	(0.044)
logphosphate	0.091***	0.091***	0.062***	0.065***	0.083***
	(0.013)	(0.013)	(0.017)	(0.017)	(0.018)
logtotlabor	0.243***	0.243***	0.202***	0.182***	0.212***
	(0.032)	(0.032)	(0.04)	(0.047)	(0.042)
logarea	0.451***	0.451***	0.5***	0.494***	0.395***
	(0.035)	(0.035)	(0.064)	(0.083)	(0.081)
dp	0.034	0.034	0.031	0.037	0.071
	(0.032)	(0.032)	(0.039)	(0.048)	(0.057)
dss	0.053**	0.053**	0.002	-0.01	-0.005
	(0.021)	(0.021)	(0.024)	(0.023)	(0.027)
dv1	0.178***	0.178***	0.212***	0.187***	0.138**
	(0.041)	(0.041)	(0.057)	(0.062)	(0.067)
dv2	0.174***	0.174***	0.074	0.157	0.173*
	(0.057)	(0.057)	(0.101)	(0.118)	(0.101)
dr1		-0.035	-0.036	-0.072	-0.041
		(0.052)	(0.06)	(0.069)	(0.086)
dr2		-0.022	-0.168	-0.254*	-0.246*
		(0.106)	(0.112)	(0.142)	(0.148)
dr3		0.001	-0.106	-0.205	-0.177
		(0.11)	(0.122)	(0.15)	(0.159)
dr4		0.124	0.096	0.024	-0.006
		(0.085)	(0.082)	(0.105)	(0.114)
dr5		0.12	0.007	-0.05	0.011
		(0.108)	(0.122)	(0.147)	(0.157)
mlogseed		0.042	0.026	0.089	0.005
		(0.074)	(0.075)	(0.082)	(0.106)
mlogurea		0.096**	0.006	-0.008	0.005
		(0.04)	(0.055)	(0.061)	(0.072)
mlogphosphate		-0.068**	0	-0.014	-0.018
		(0.03)	(0.033)	(0.037)	(0.039)
mlogtotlabor		-0.08	-0.162**	-0.126	-0.02
		(0.073)	(0.073)	(0.083)	(0.083)
mlogarea		0.058	0.196**	0.113	0.105
		(0.079)	(0.094)	(0.114)	(0.135)
mdp		-0.082	-0.147*	-0.166*	-0.219***
		(0.071)	(0.079)	(0.09)	(0.082)
mdv1		-0.057	-0.128	-0.172	-0.015
		(0.109)	(0.125)	(0.152)	(0.166)
mdv2		-0.149	-0.163	-0.021	-0.061
		(0.148)	(0.193)	(0.194)	(0.186)
Mean Technical Efficiency	0.569	0.629	0.769	0.729	0.67

Standard errors in parentheses
***,**, & * refer to significance at the 1%, 5%, and 10% levels, respectively

Figures

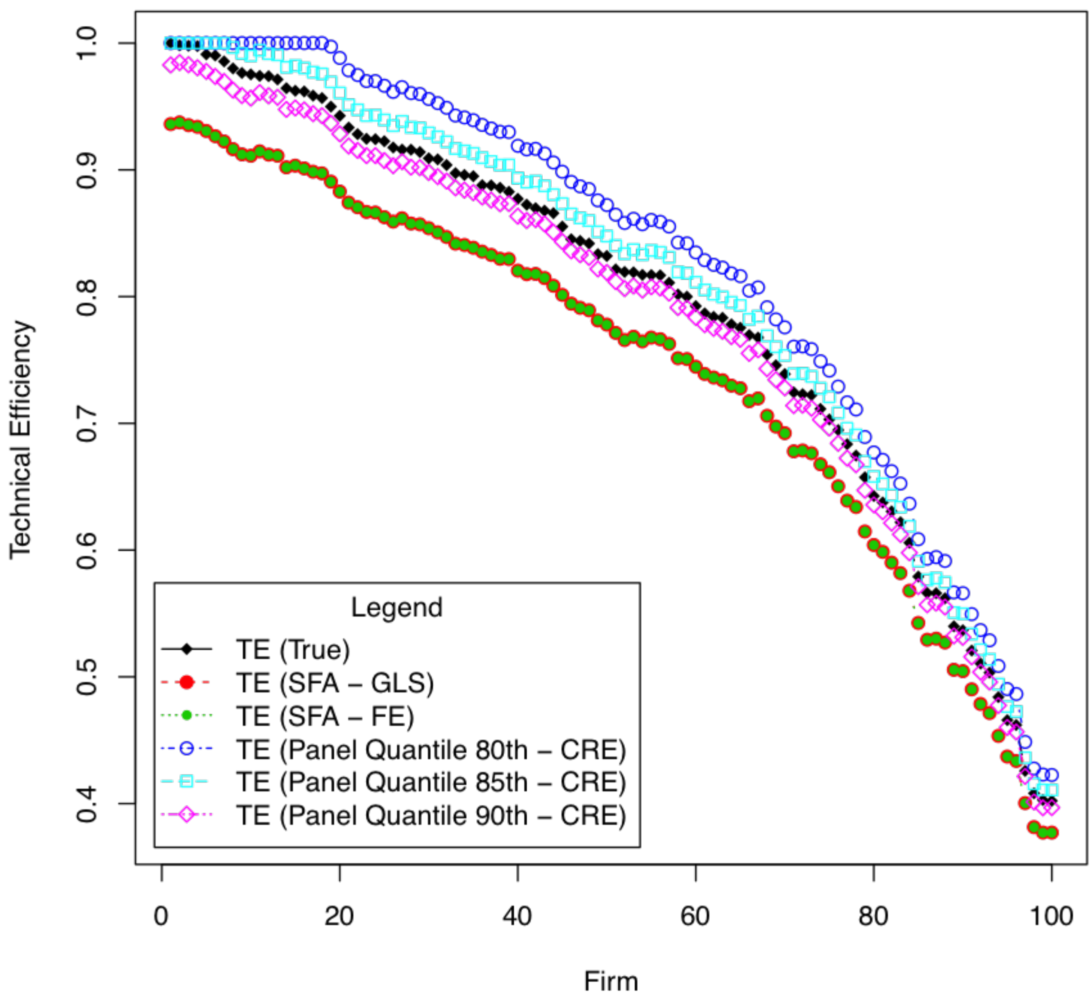

Figure 1: Technical Efficiency Scores - One Firm Fully Efficient, Explanatory Variables Uncorrelated with Inefficiency Term

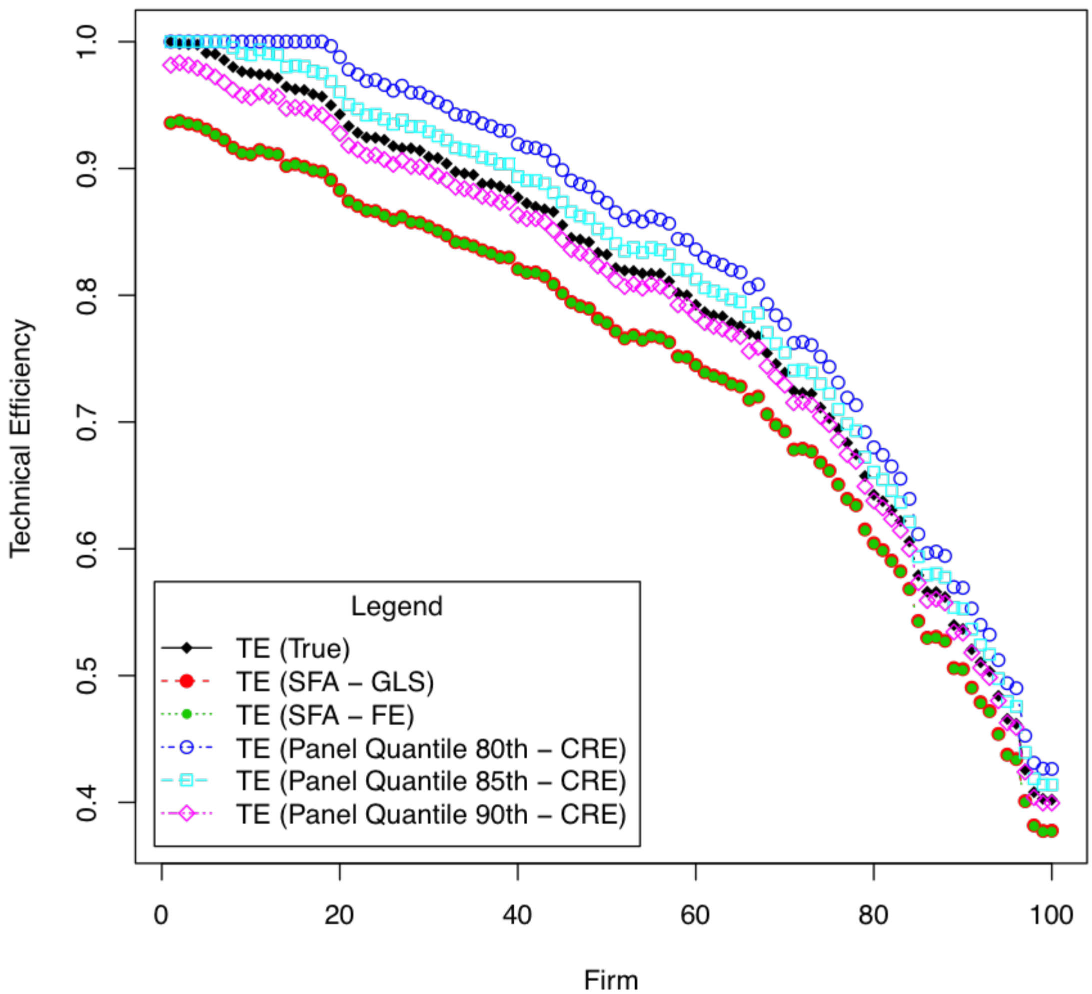

Figure 2: Technical Efficiency Scores - One Firm Fully Efficient, Explanatory Variables Correlated with Inefficiency Term

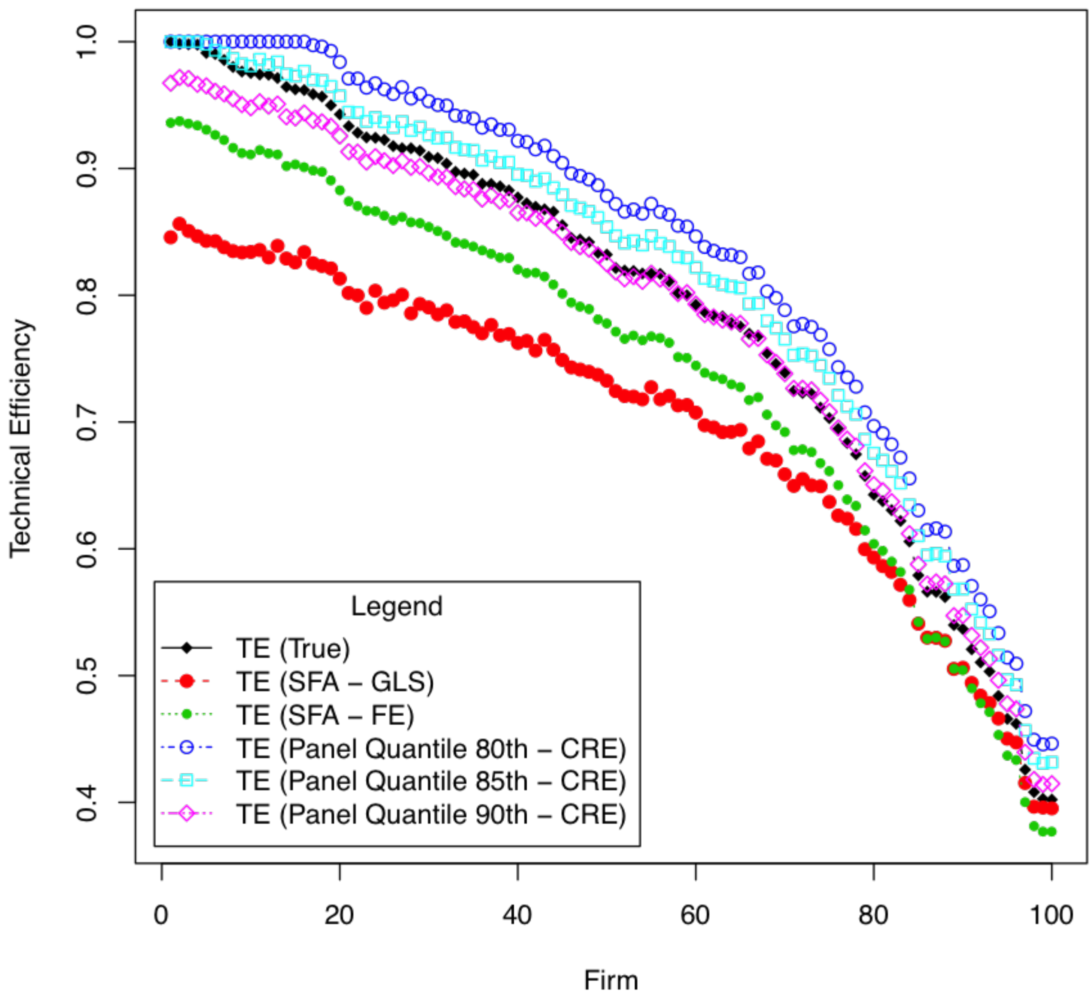

Figure 3: Technical Efficiency Scores - One Firm Fully Efficient, Explanatory Variables Correlated with Inefficiency Term, Mundlak Specification

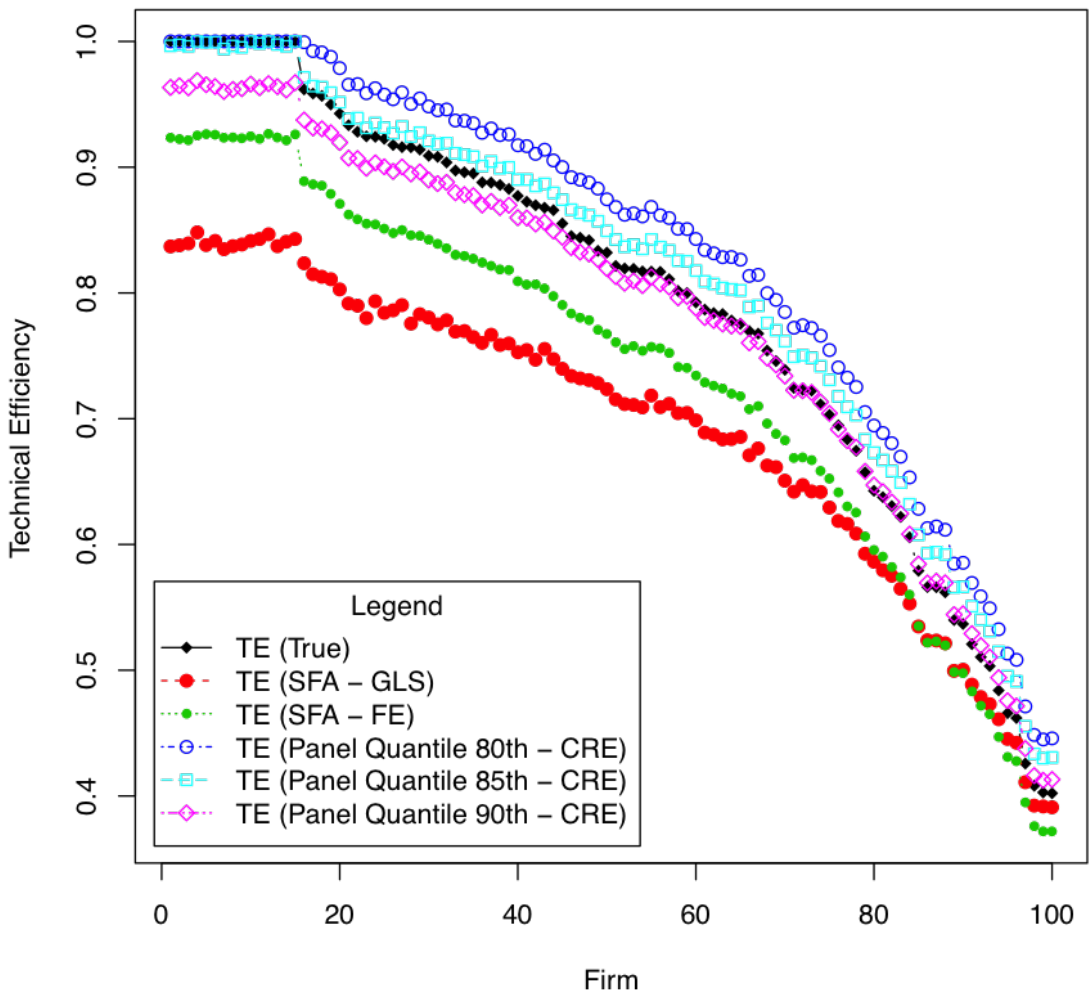

Figure 4: Fifteen Firms Fully Efficient

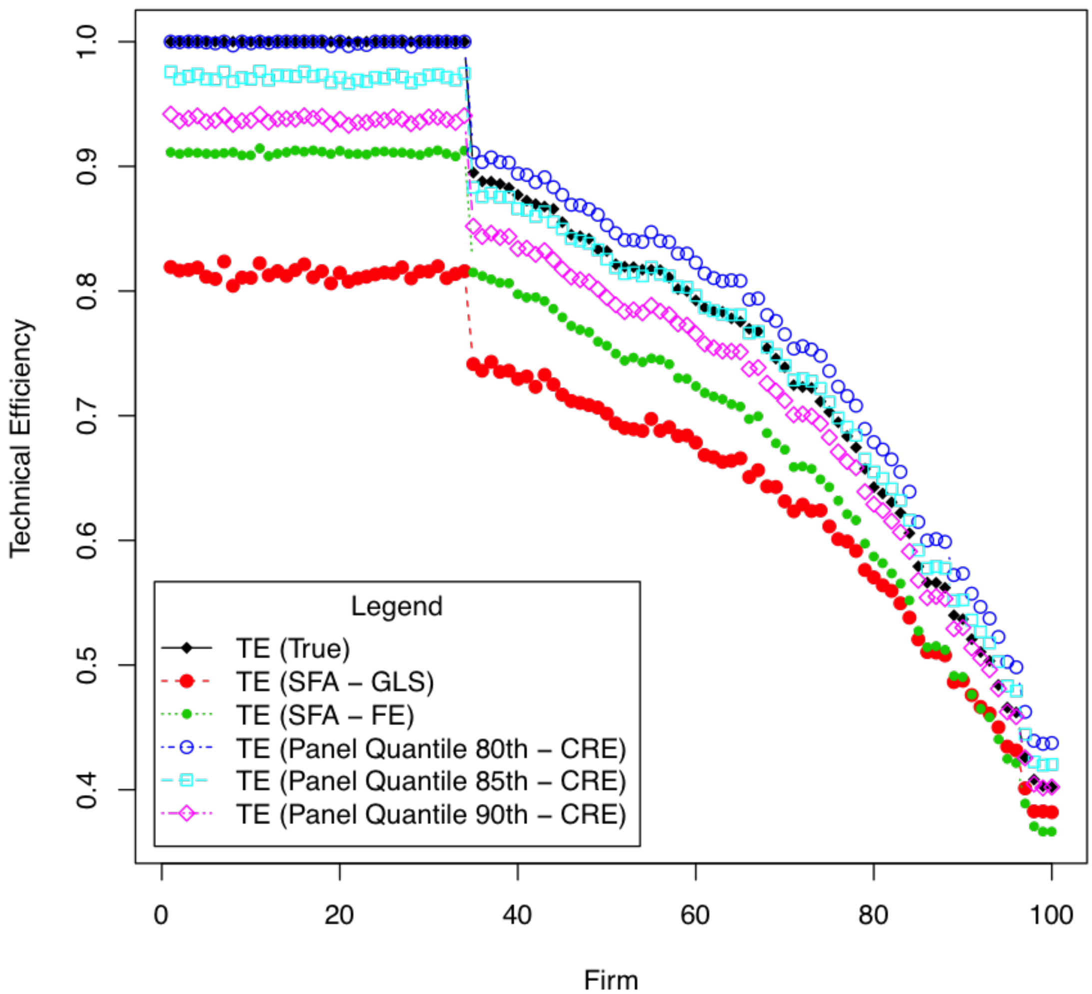

Figure 5: Thirty-four Firms Fully Efficient

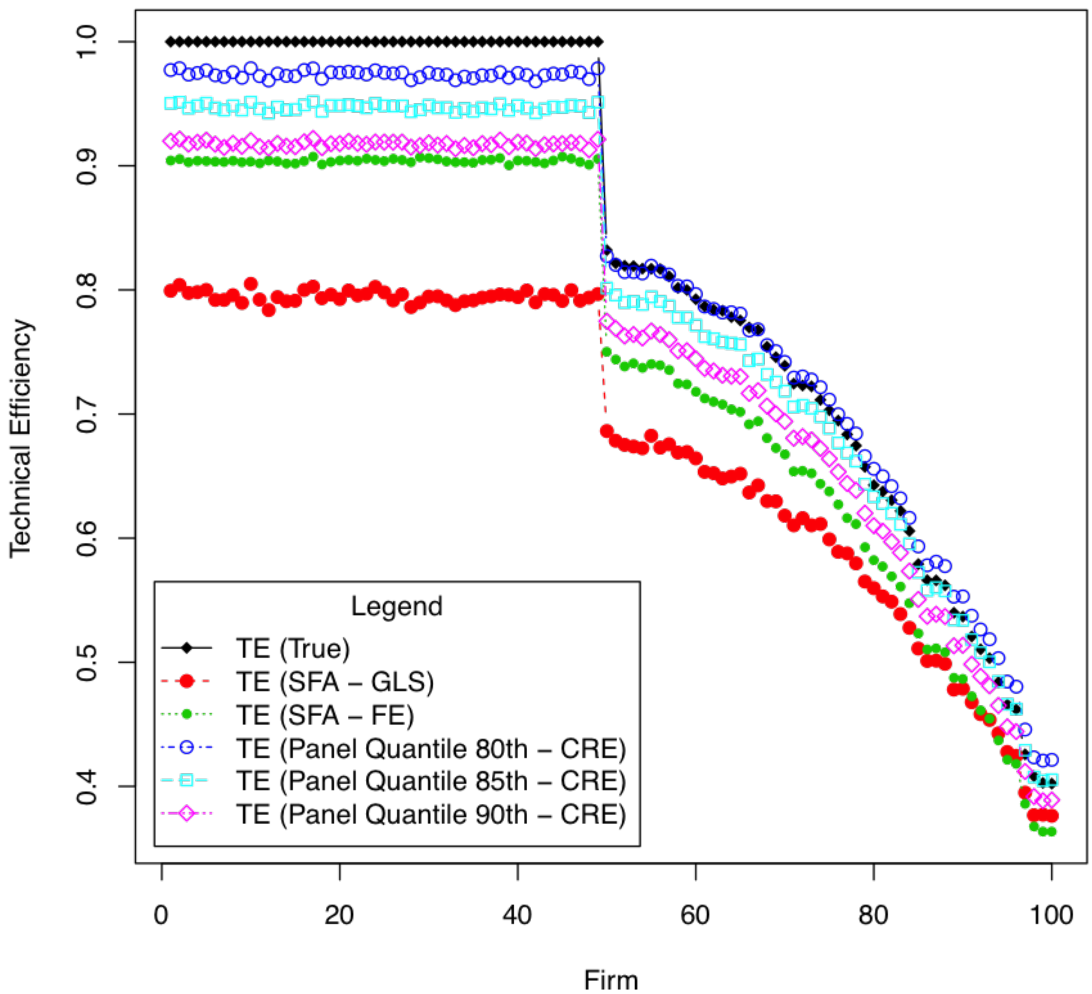

Figure 6: Forty-Nine Firms Fully Efficient

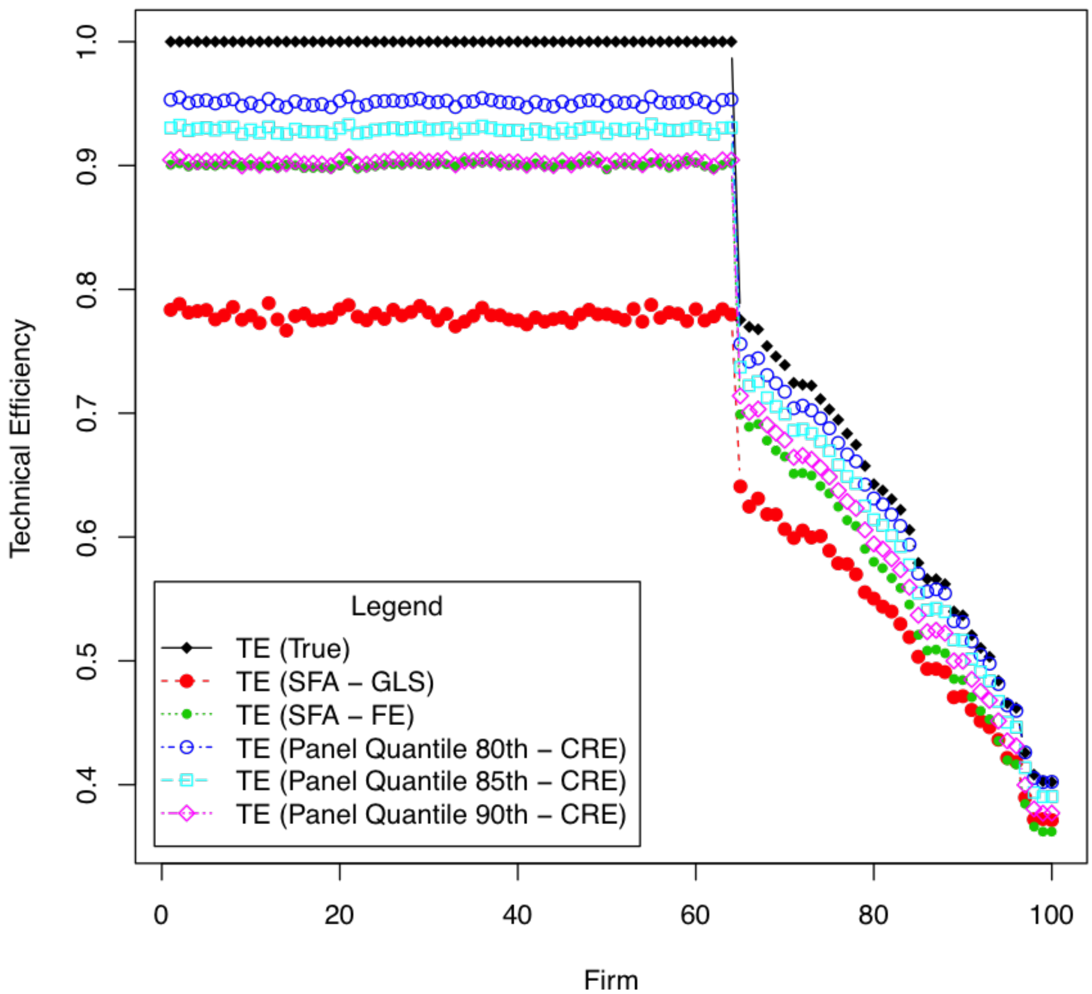

Figure 7: Sixty-four Firms Fully Efficient

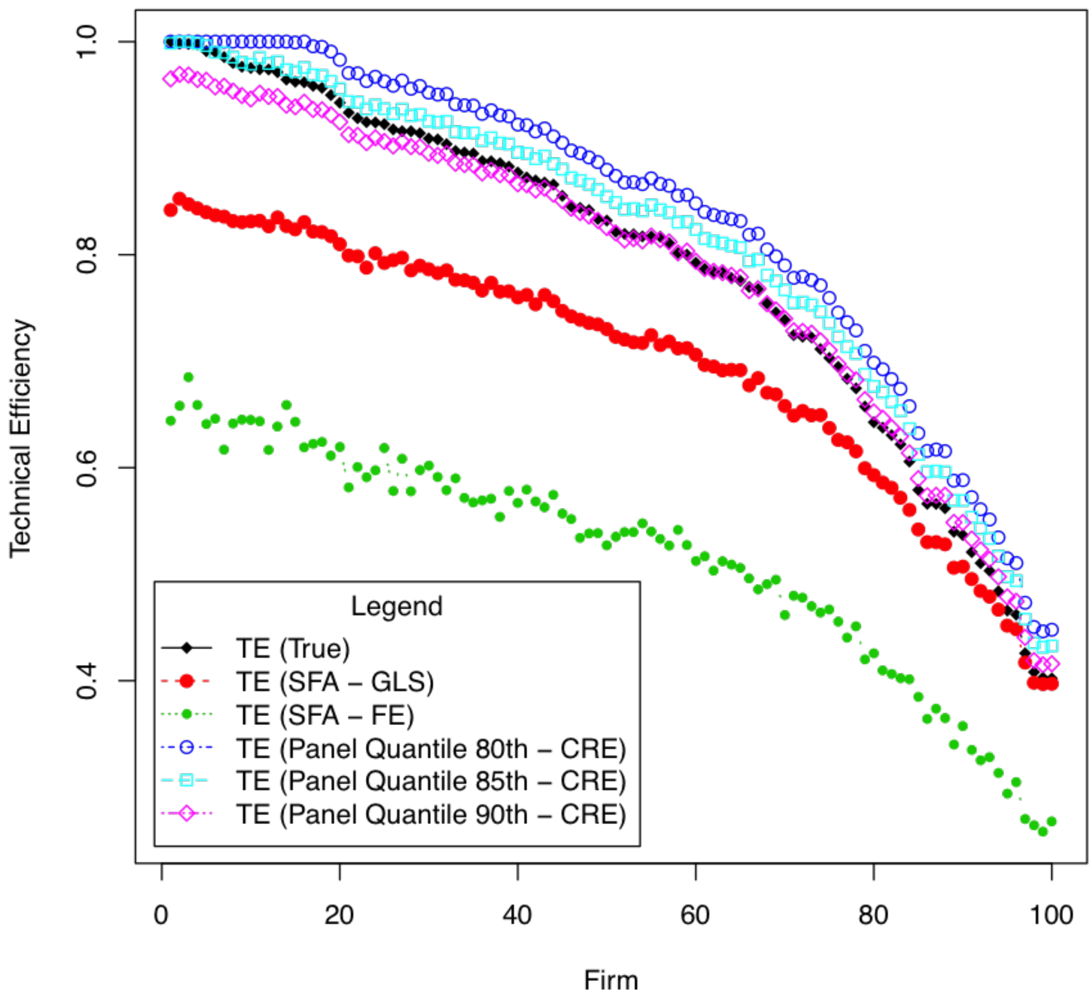

Figure 8: One Firm Fully Efficient, 1 Time-Invariant Variable in Production Process

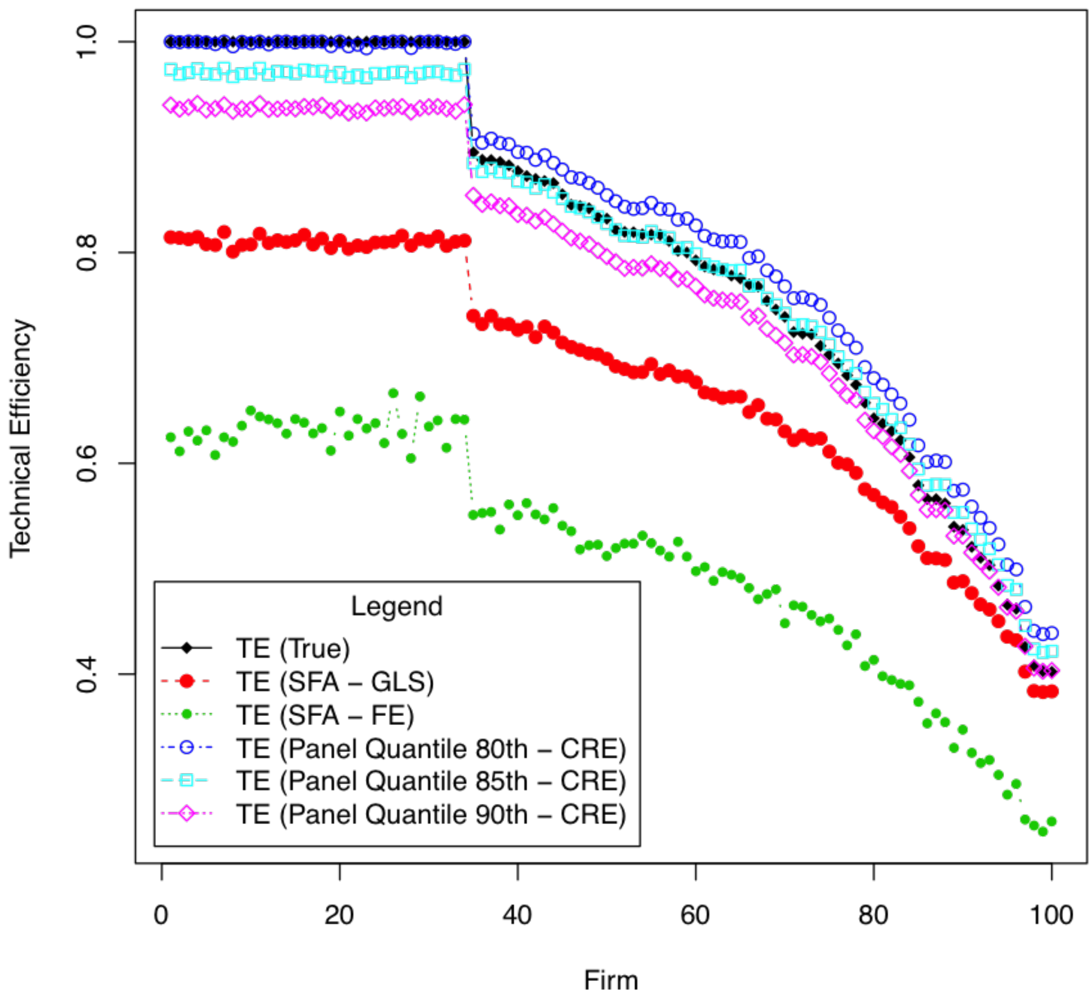

Figure 9: Thirty-Four Firms Fully Efficient, 1 Time-Invariant Variable in Production Process

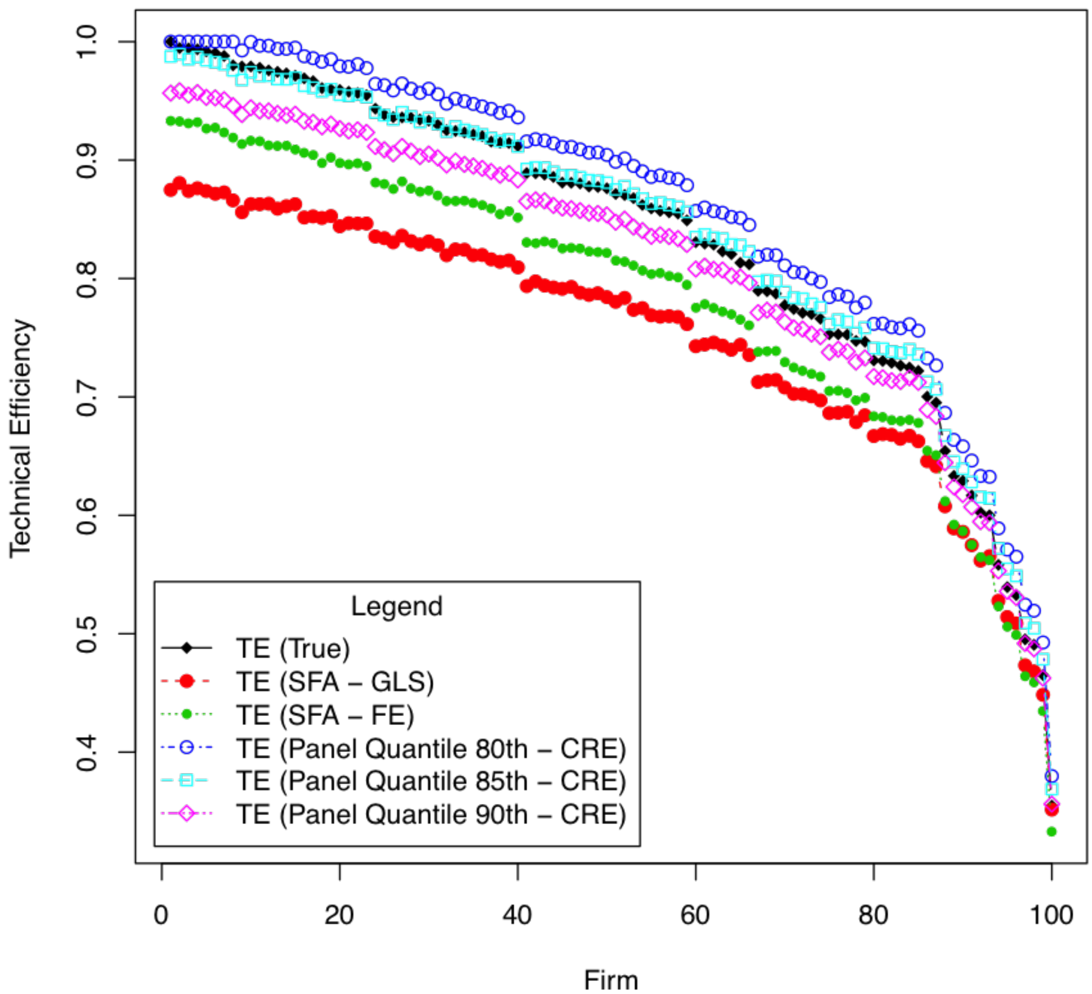

Figure 10: One Firm Fully Efficient, Exponential Inefficiency Term

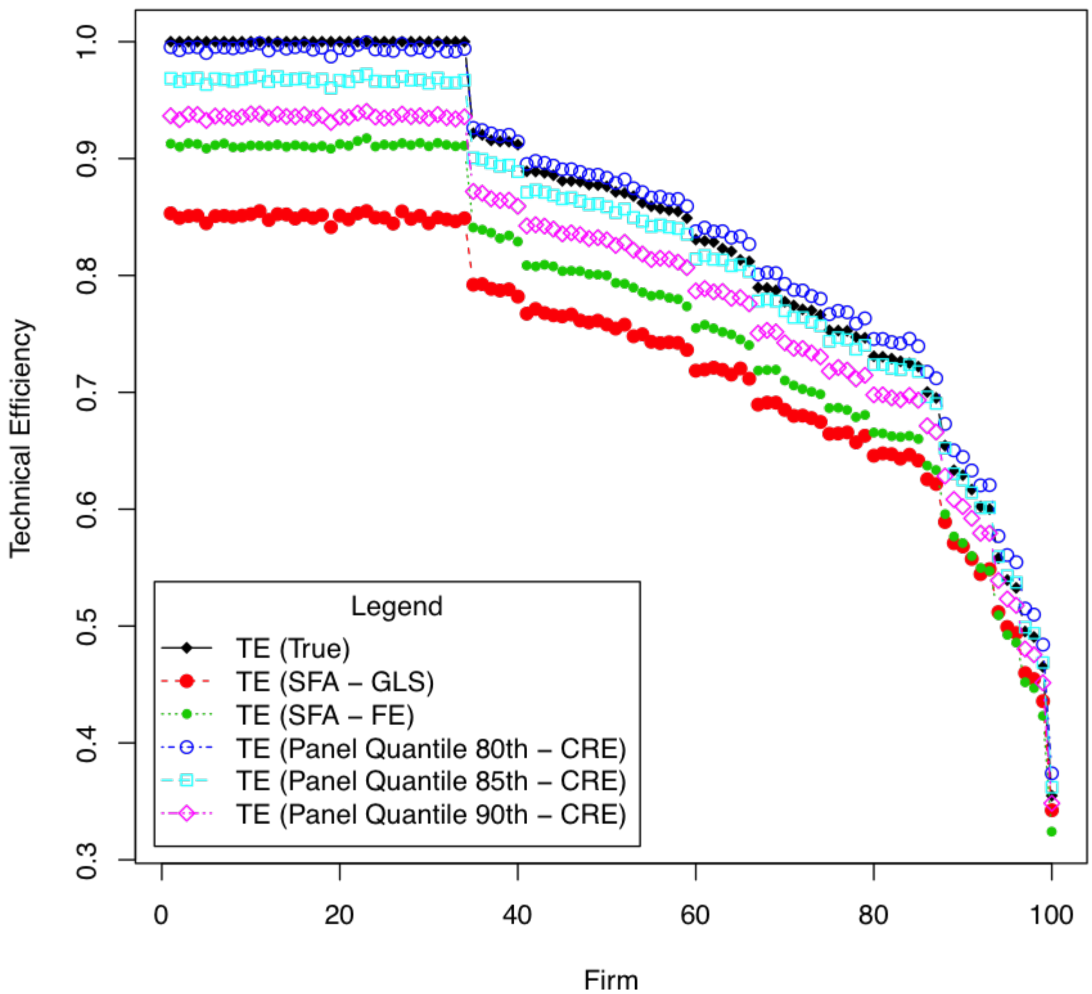

Figure 11: Thirty-Four Firms Fully Efficient, Exponential Inefficiency Term