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THEORETICAL SIMULATION IN HEALTH ECONOMICS: AN APPLICATION TO GROSSMANS MODEL OF INVESTMENT IN HEALTH CAPITAL

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Theoretical Simulation in Health Economics: An application to Grossman's Model of Investment in Health Capital

Abstract

In this paper we argue on the merits of theoretical simulation, a widely used technique in other areas of theoretical economics, to extending the phase diagram analysis of the Grossman's 1972 model of investment in health capital. We argue that theoretical simulations are particularly useful when considering problems with multiple state variables. To illustrate, we perform simulations with varying assumptions of health depreciation rates and conditional survival probabilities and generate time-plots for the evolution of health capital and health investments over individual finite lifetimes.

JEL Classification: I12; C61; C63

Key words: theoretical simulation; grossman model; health capital

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1. Introduction

Michael Grossman's (1972) model of investment in health capital is the standard model for the analysis of health related behaviours and the consumption of health-related commodities. Its appeal rests on its explicit recognition of the dynamic nature of the problem, and the way it allows decisions about health-related behaviours to be framed as part of an intertemporal optimization problem. In particular, when it is set up in the form of an optimal control problem it looks at the intertemporal aspect not just in terms of adjacent periods but from a lifetime perspective.

There are a number of alternative approaches to modelling intertemporal optimization decisions: they can be set up as discrete time optimization problems, as in Grossman's original formulation, as dynamic programming problems, or as optimal control problems. All will give the same fundamental solution to the problem, but the way the necessary conditions for a solution are presented differs across methods. Optimal control theory, which we take as our starting point here, has the appeal of allowing us to draw phase diagrams in either state-control or state-costate space. Drawing a phase diagram allows us to look at a graphical representation of the lifetime trajectory which falls out of the individual's problem, where her initial state of health is one of the conditions underlying the solution, and a terminal condition, either on the terminal stock of health capital or on the terminal marginal value of health capital is derived as part of the solution process. The combination of the initial and terminal conditions along with the rules which determine the level of health investment at each instant of time and the way the marginal value of a unit of health capital changes over time serve to tie down which of the candidate optimal health investment trajectories is optimal for the individual whose behaviour we are modelling.[1]

One point which the use of the phase diagram technique helps clarify is that, when we are dealing with a finite-lived individual, the usual concept of equilibrium makes no real sense.[2] The phase diagram for the Grossman problem contains an equilibrium point, but

[1]Our discussion here is in terms of a bare-bones version of the Grossman model: see Laporte (2014), for example.

[2]There is debate in the literature about whether the Grossman model allows an individual to choose to live forever. While there are points of technical interest in this debate, we take as given here that, as a

that point will be approached only over an infinite horizon. In a finite horizon problem we can define an optimal lifetime trajectory along which the necessary conditions for lifetime utility maximization are always satisfied, but no equilibrium, in the sense of a point which has the property that, once we have reached it there is no intrinsic tendency to depart from it.[3] As we usually think of an equilibrium point in static analysis, it is a function of the exogenous variables of the problem and, so long as those exogenous variables remain unchanged, so too will our endogenous variables.

In dynamic analysis, this way of thinking about the equilibrium is useful for infinite horizon problems but not for finite horizon ones. For a finite horizon problem there will be an optimal trajectory whose position will depend on the values of the exogenous variables, but the fact that the exogenous variables remain unchanged does not mean that the endogenous ones also remain unchanged, rather it means that the location of the optimal trajectory on the phase diagram will not change, but the endogenous variables-in our case health investment and health capital-will change over time as the individual follows her optimal lifetime trajectory. In effect, health capital and health investment are trended goods, where the trend solved as part of the problem may well be intrinsically non-linear and does not involve a constant value of change (or rate of change) per unit of time. Thus simply adding a trend term is not likely to capture the fine detail of the evolution of an individual's health capital, nor is a partial adjustment framework, of the form used by Wagstaff (1986) likely to suffice. It is certainly true that changes in the exogenous variables of the problem will cause the optimal trajectory to shift, but a proper understanding of the nature of the trajectory is essential as a starting point for empirical analysis.

2. Two state variable problems

As we noted above, the appeal of the optimal control methodology lies in part in the fact that the phase diagram methodology gives us at least a qualitative picture of the individual's lifetime health investment behaviour. The weakness of the phase diagram

rational, forward looking human being, our optimizing individual knows that her life will be finite.

[3] In economic modelling we most often assume that we will reach it-i.e. that it is dynamically stable. In saying this we are implicitly assuming that it exists, existence being a separate matter from existence.

approach is that it can only be used in the case where the problem contains a single state variable-in the Grossman model, health capital. Grossman's original (1972a) formulation in principle contained two state variables, health capital and financial capital, although in his formalization of the problem Grossman introduced financial capital via a lifetime budget constraint. Ehrlich and Chuma (1990) set up the Grossman model in two-state variable form, with an equation of motion for financial capital.

In fact, Grossman's (1972b) formulation of the problem contained three state variables, since much of his discussion in that paper concerned the effects of allowing the rate of depreciation of health capital, δ, which was initially treated as a constant, to increase as the individual ages. Since the derivative of age with respect to time is necessarily equal to 1, this serves to introduce an equation of motion for δ, which turns it into a state variable. Thus even if we do not introduce an equation of motion for financial capital, making δ a function of time converts the problem from a one-to a two-state variable problem, which cannot be phase diagrammed. It is this version of the problem on which we will focus most of our attention in what follows.

Grossman originally introduced the notion of δ increasing over time as a way of tackling the length of life problem. Since he did not impose a finite horizon on the problem, he was faced with the possibility that his decision maker could indeed choose to live forever. As Ehrlich and Chuma (1990) note, when we are working in continuous time terms, there is a transversality condition which endogenizes the terminal value of T-essentially establishing conditions under which an individual who might in principle be able to live forever will optimally choose instead to die after a finite life-span. As Ried (1996) notes, there is no corresponding transversality condition for a discrete time optimization problem, but it should be noted that Grossman's approach was consistent with the spirit of the continuous time transversality condition, in that the optimal end of the life-span occurred when the marginal cost of continuing to live rose to equal the marginal benefit of life extension, so that to live any longer would involve the marginal cost of longer life exceeding its marginal benefit. Grossman achieved this by making the rate of depreciation of health capital h increase over time, causing the amount of investment necessary to prevent h falling below $hmin$ (the level of h at which death occurred) to rise, and with it the cost of staying alive. Thus Grossman made δ a function of time for a very specific purpose, and it is in that context, rather than as an illustration of a two state variable problem, that most of the literature (Ehrlich and Chuma (1990), for example) has considered it.

Nevertheless, the analytical problems associated with a two-state variable control problem have made themselves manifest in this strand of the health capital literature. Intuitively it is fairly clear how making δ increase through the individual's life course is likely to affect her health investment decisions. If we consider an individual who is born healthy, we will in general find that her optimal trajectory involves her stock of health capital declining through her life course, but not at a constant rate. h declines because of the operation of δ, and the role of health investment, z, can be thought of as to slow the natural rate of decline from the natural rate to an optimal rate.[4] As the rate of depreciation rises, later in life, h will tend to decline faster, for most individuals at a rate which is notably faster than the optimal rate of decline, so the optimal value of z will tend to increase by whatever amount is necessary to slow the decline in h to a rate which is optimal, given the costs and benefits involved.

It is not, however, easy to validate that intuition theoretically. As we have already noted it is in general not possible to draw a phase diagram for a two-state variable control problem.[5] It might be thought that, given the way in which δ affects the phase diagram for the basic Grossman model, we might introduce an increasing δ by rotating the stationary loci for the problem, causing the location of the equilibrium to shift over time. There are, however, a couple of fundamental problems with this notion. One is that, absent a time axis, we have no basis for deciding how rapidly the loci should shift, nor how we should represent that on the diagram. We might consider approximating the effects of a rising rate of depreciation by assuming that δ takes on two values, a lower one when the individual is young and a higher one when she is older. Instead of a continuous change in the locations of the stationary loci there will be a discrete change at the point in time at which the increase in δ takes effect. While we can analyze this approximation to the two-state variable problem using a phase diagram, since there are well-established transversality conditions for linking the two stages of a two-stage control problem, there is a problem in

[4]For the case of an individual who is born unhealthy, see Laporte (2014).

[5]See, for example, Pitchford (1977) and Leonard and Long (1992).

that both stationary loci will shift, but not in a manner which yields any determinate change in the location of the equilibrium for the problem and, as we have already noted, the individual will not be heading towards the equilibrium in any event, so we have the additional problem of deciding how to represent a shift in the optimal trajectory when the equilibrium shifts in an indeterminate manner. Thus we cannot modify our qualitative analysis in a simple manner to incorporate what is basically a pretty simple modification of the problem (certainly simpler than adding an equation of motion for financial capital).

Because of its prominence in making Grossman's optimizing individual finite-lived, the case of a time-varying δ has been the subject of a certain amount of formal analytical work. The general approach of this work has been that of the dynamic envelope theorem. In static analysis the envelope theorem refers to the fact that if we differentiate the optimized Lagrangean expression for a problem with respect to one of the exogenous variables of the problem, the result can be interpreted as the shadow price of that variable and the derivative of the maximized objective function with respect to that variable. Hence the definition of the Lagrange multiplier on the consumer's budget constraint in a utility maximization problem as the marginal utility of income. Similar exercises can be undertaken in dynamic analysis where the interpretation is in terms of impact on the maximized value of the intertemporal objective function. While the static envelope theorem frequently provides useful guidance for empirical work, dynamic envelope theorem results are seldom as fertile, because of the intertemporal nature of the maximized objective function. In applications of the dynamic envelope theorem approach to the case of an increasing rate of depreciation of health capital it is common to look at the effect of a one-shot increase in δ, rather than trying to analyze a continuously changing rate of depreciation.

Thus Eisenring (1999) applied the comparative dynamics approach in Oniki (1973) to investigating the effects of a change in an exogenous variable to the rate of depreciation of health capital in Bruce Forster's (1989) version of Grossman's model.[6] His results contain a considerable amount of indeterminacy with regards to the effect of an increase in δ on the optimal trajectories.[7]

[6] See Forster (1989).

[7] Eisenring's results are to some degree consistent with our earlier heuristic discussion of the effect of a one-shot change in δ in a phase diagram, where we said that the fact that both stationary loci would shift

Ried (1996) and Ried (1998) consider in some detail the effect of a change in δ on the optimal trajectory. Ried (1998) uses a discrete time formulation which allows him to break the effect of a change in δ into its effect on individual periods before and after the change, and adopts what is in effect a dynamic programming perspective, starting by considering the effect of the increase when it occurs in the final period. While Ried's approach yields more detail than Eisenring's it can only be described as labourious, and it is not clear that either approach is really all that much more informative than assuming a single change in δ and drawing the phase diagram for a two-stage optimal control problem. In addition, Eisenring notes that Oniki's technique is not feasible when there is more than one state variable, which means that it could not be applied to Grossman's original formulation when written in two-state variable form. We suggest that, even for something as relatively straightforward as making δ our second state variable, theoretical simulation would seem to be the preferred approach to extending phase diagram analysis.

3. Theoretical Simulation Analysis

Theoretical simulation is widely used in macrodynamic economics, in which the microfoundations are taken to derive from intertemporal optimization decisions of individual economic agents. Those models commonly adopt a representative agent approach, and set the macroeconomy up as an infinite horizon optimization system which, because it is basically an optimal control problem, has a saddlepoint equilibrium. The infinite horizon assumption simplifies the simulation of the control problem to a certain degree since the transversality conditions for an infinite horizon problem make the stable branch to the equilibrium the optimal trajectory, and the equilibrium is a meaningful point so that it makes sense to write a policy function, with the control variables as functions only of the exogenous variables of the system. The policy function is in essence a device for comparative static analysis since it shows how the equilibrium value of the endogenous variables changes

(and given the nature of the shifts) made the impact of the shift on the optimal finite horizon trajectory qualitatively indeterminate. To some degree, Wagstaff (1993) empirical results are also consistent, when Wagstaff divided his sample into two broad age ranges, the younger of which was presumed to have a low value of δ and the older a high value. This effort did not produce particularly sensible looking results.

in response to changes in the exogenous variables. Knowing how the exogenous variables and the equilibrium values of the endogenous variables change makes it easier to determine the optimal trajectory from the old to the new saddlepoint equilibrium.

Theoretical simulation has been used in the health capital literature before, most notably by Martin Forster (2001).[8] Forster used simulation techniques to consider the effects of the various conditions which had been used to define death in a model which was built on the assumption that individual lives were finite. Briefly, if we take the end of the planning horizon to be a fixed, finite point in time, we have a number of possible transversality conditions from which to choose. The simplest is to define a terminal health stock $hmin$ and require that the individual end up at exactly that level of h at exactly T, and not before. An alternative transversality condition would be to run h down to zero at precisely T, but this is generally not possible with a constant rate of depreciation of health capital, and the hard $hmin$ condition can be seen as a reasonable alternative. Here $hmin$ can be seen as Grossman's "death" stock of health capital.

A third possibility, often used when the state variable cannot be run down to zero, is that the costate for the problem (the shadow price of health capital) reach zero at exactly T, meaning that, whatever the individual's actual value of h at time T, another unit would yield her no utility. Alternatively we could define $hmin$ as a lower bound to acceptable health; a level which the individual is not willing to fall below at any point in her life but would be willing to fall to at time T. If the optimal trajectory has h reaching $hmin$ at T no further transversality condition is needed, if optimality involves being healthier than $hmin$ at T then the costate must be zero at that point in time. Forster uses theoretical simulation to investigate the implications of these various conditions for the optimal trajectory, and we have discussed them here because, while they are not the focus of our simulations, we shall see that the choice of terminal condition does play a significant role in determining the shape of the individual's optimal lifetime health capital trajectory.

While Forster uses theoretical simulation, he works within the constraints of a one-state-variable problem and indeed, uses simulation techniques to plot phase diagrams for

[8]More recently, Carbone and Kverndokk (2014) use simulation techniques to examine factors that influence individual health and education investments and their empirical correlation.

the various cases of terminal transversality conditions which he considers. This means that his approach does not escape the problems which the two-state variable problem poses for qualitative phase diagram analysis. We propose to extend the use of theoretical simulation by adopting the approach of the macrodynamic simulation literature and generate time-plots for health capital and health investment. Thus we will be able to find solution trajectories for the individual-s lifetime optimization problem even in the case where we make δ into a state variable. The basic structure of the model is described below.

The optimizing individual's problem can be written as:

$$max \int_0^T U\left(x,c,h\right)e^{-\rho t}dt, U_i > 0, U_{ii} < 0, i = x,c,h \tag{1}$$

where x is a general consumption good which yields utility but has no impact on the individual's health, c is a good (cigarettes, perhaps) which yields utility from consumption but is harmful to one's health, h is the individual's stock of health capital and ρ is the individual's subjective rate of discount. We also define a health investment good z which yields no utility in itself but has a positive effect on the individual's health. Maximization is subject to the equation of motion for health capital:

$$\dot{h} = g(z) - f(c) - \delta h \tag{2}$$

where $g(z)$ and $f(c)$ are separable elements in the health production function, $g_z > 0$, $g_{zz} < 0$, $g(0) = 0$, $f_c > 0$, $f_{cc} > 0$, $f(0) = 0$, and $\delta > 0$ is the rate of depreciation of health capital.

The individual's spending is subject to the instantaneous budget constraint:

$$y = p_z z + p_c c + p_x x \tag{3}$$

Here y is current income, which for simplicity here we do not make a function of h (thus we are dealing with the consumption version of the Grossman model-extension to the investment version adds no issues which are relevant to our topic) p_z is the market price of health investment goods and p_x is the market price of consumption goods, which we at this point we shall normalize to 1, so that income is in real terms and the other two prices are relative prices, and p_c is the market price of goods which are bad for one's health. Note that we are assuming that the budget constraint is binding at all t, so there is no saving or borrowing in our model. This is just a simplifying assumption which allows us to focus on the effects of adding an equation of motion for δ. It also lets us eliminate x from the problem.

The Hamiltonian for this version of the problem is:

$$\mathcal{H} = U(Y - p_z z + p_c c, c, h) + \psi(g(z) - f(c) - \delta h) \tag{4}$$

In a discrete time optimization problem we use Chow's Lagrangean approach to intertemporal optimization.[9] The Lagrangean for the problem and first order conditions are depicted in the appendix of the paper. As in the continuous time phase diagram analysis we work in a fixed, finite horizon setting with a fixed endpoint, where the terminal value of h is set at $hmin$.

The particular functional forms used in our simulations are:

$$U(x_t, c_t, h_t) = x_t^{\alpha_1} c_t^{\alpha_2} h_t^{1-\alpha_1-\alpha_2} \tag{5}$$

$$f(c_t) = \eta c_t^b \tag{6}$$

$$g(z_t) = \gamma z_t^a \tag{7}$$

The specific parameter values are given in table 1:

Table 1: Parameter Values

δ	η	γ	a	b	α_1	α_2	p_z	p_x	p_c	β	h_0	$hmin$	T	d_0	d_1	d_2
0.12	1	0.5	0.6	2.5	0.32	0.32	1	1	1	0.99	1	0.5	12	-2	0.015	0.0015

We solved the individual's optimization problem using techniques which are common in theoretical macroeconomic dynamic models, assuming a finite horizon of 12 periods. In the present case we are treating y as exogenous for our individual. Also, since the initial value of health h_0 is significantly above hmin, we take it as representing an individual who is in good health at birth. We display health investment choices in the form of the ratio of z, which represents healthy activities, to c, which represents unhealthy activities. We investigate two primary modifications to the Grossman (1972b) model; our main one in

[9]See Ferguson and Lim (2003) Dynamic Economic Models in Discrete Time, Routledge, London and New York.

which the rate of depreciation of health capital increases continuously over time, and a second in which we introduce the effect of age-specific survival probabilities. The solution algorithm is described in the Appendix.

3.1. Baseline Analysis, constant δ

The first of the simulations which we report here is a baseline run, holding the rate of depreciation constant and holding the individual's survival probability constant at 1 in each period, so that there is no element of uncertainty about whether she will reach the end of the planning horizon. In Figure 1 below we show the z/c ratio for an individual who was born with an initial health stock h_0, and in Figure 2 we show her lifetime health capital trajectory.

Figure 1: Lifetime z/c Ratio Trajectory

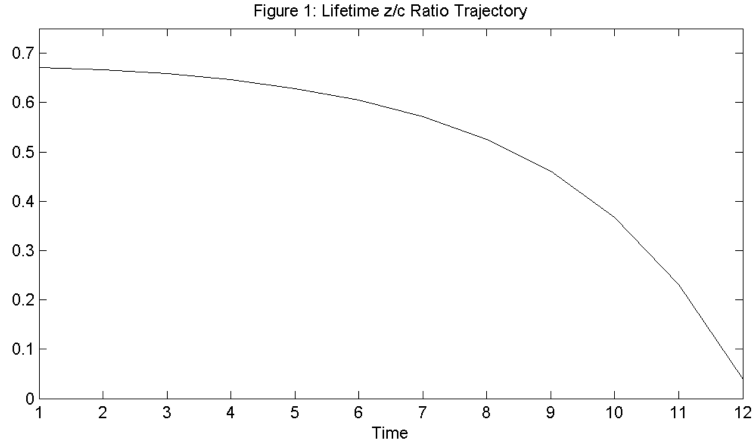
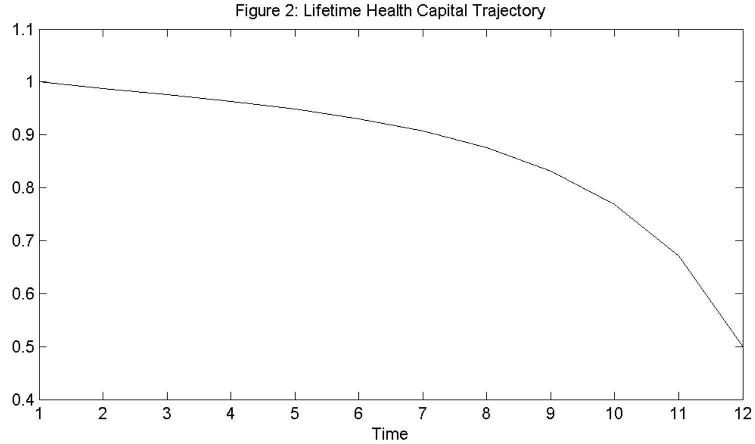


Figure 2: Lifetime Health Capital Trajectory



In considering these graphs we should note the importance of the transversality condition as imposed in the theoretical simulation: when we are working with a fixed endpoint problem the final values of health investment will be determined by the requirement that the terminal value of h take on a specific value. Theoretical simulation can obviously be used to investigate the implications of alternative transversality conditions, in particular the case which we noted earlier in which the terminal value of h is a lower bound and the costate equals zero when the optimal trajectory is going to end at a value above that lower bound. Figure 1 shows that health investment (defined here as the z/c ratio) remains relatively constant the first part of life, as does h (as shown in Figure 2), and that both start to fall off more noticeably in the last few periods.

Figure 3: Lifetime z/c Ratio Trajectory for a High and Low Income Individual

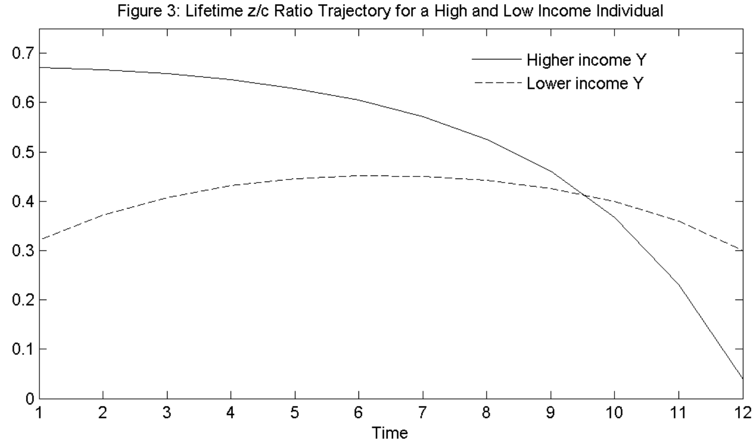
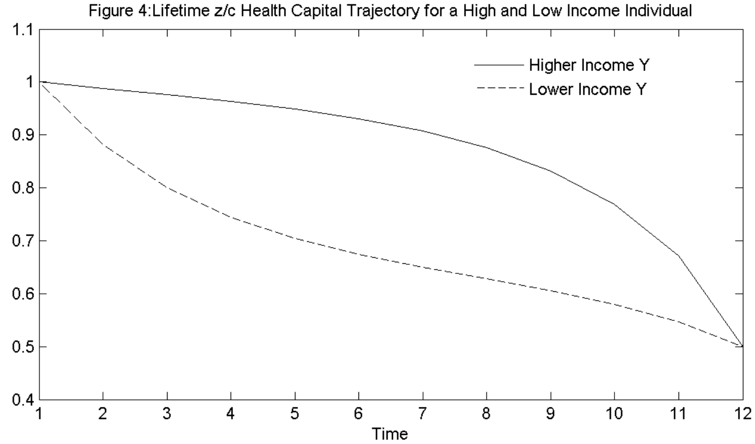


Figure 4:Lifetime z/c Health Capital Trajectory for a High and Low Income Individual



Next, staying with the model in which the rate of depreciation and the survival probabilities are constant, we consider the trajectories for two individuals, both born with the same initial level of h, but where one has a higher lifetime income than the other. We are still treating income as exogenous here, not as a function of health capital (this case was considered in a phase diagram context in Laporte (2014). We see from Figure 3 above that the higher income individual's optimal lifetime health investment trajectory lies above that of the lower income individual until the final periods of their lives, and from Figure 4 that while the two individuals start with the same initial level of h, the higher income individual's stock of health capital lies above that of the lower income individual until the last few periods, when the gap closes. The closing of the gap is a consequence of our choice to work with a fixed endpoint value of h: were we to work with a lower terminal bound on h we would expect the higher income individual to be healthier right up until T.

3.2. The case of an increasing δ

Next we turn to the case where the rate of depreciation of health capital, δ, increases over time. Since this makes the value of δ change as time passes, we have a $\dot{\delta}$ equation, making δ a state variable where:

$$\delta_t = \frac{exp\left(d_0 + d_1 t + d_2 t^2\right)}{1 + exp\left(d_0 + d_1 t + d_2 t^2\right)} \tag{8}$$

Age-specific values of δ_t are shown in Figure 5 alongside with the constant depreciation rate δ used in the baseline simulations.

Figure 5: Increasing depreciation rates

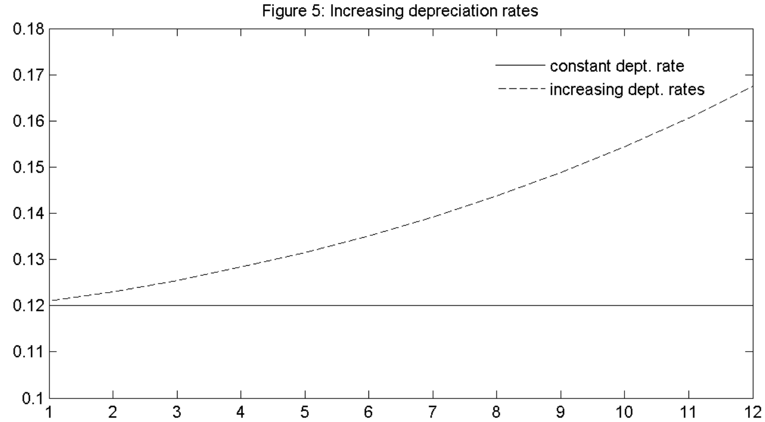


Figure 6 shows the time path of the z/c ratio for the two cases which we are comparing here, one with a constant δ and the other with δ increasing over time. In the latter case the value of δ is set equal first period of the age varying depreciation series.

Figure 6 shows that when δ increases over time (with all its values above the constant depreciation rate case) the optimizing individual's z/c ratio is lower in the earlier periods

Figure 6: Lifetime z/c Ratio Trajectory

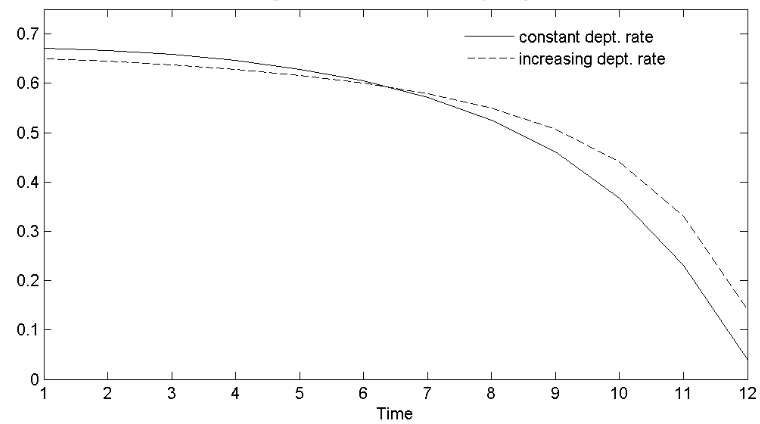
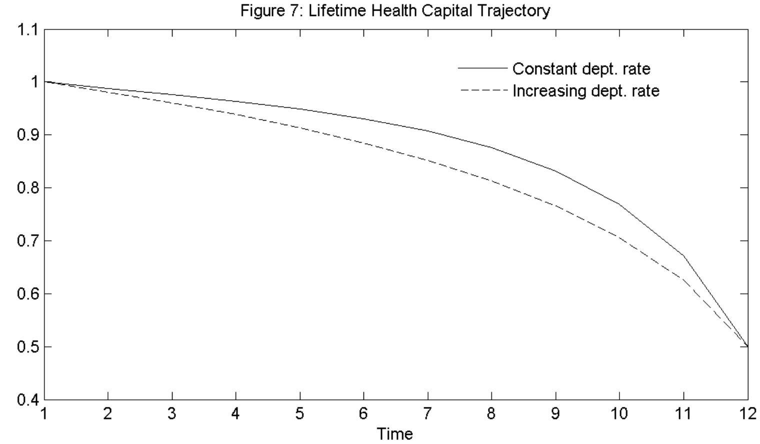
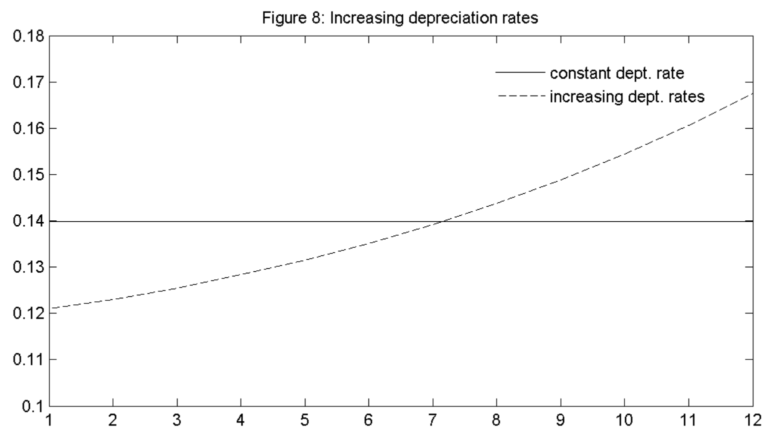


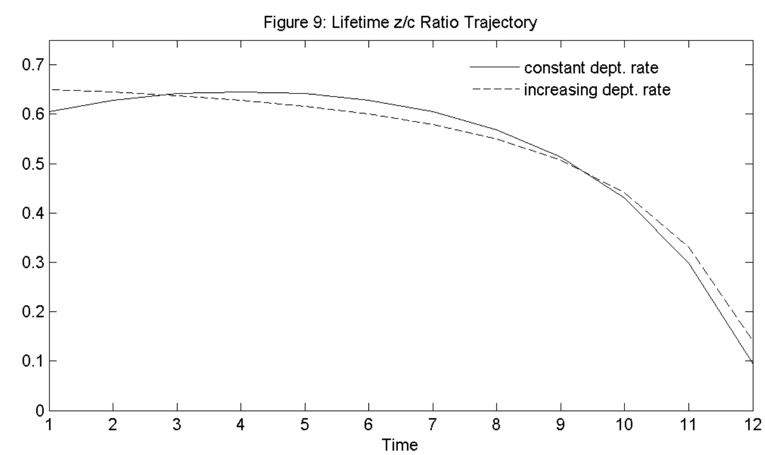
Figure 7: Lifetime Health Capital Trajectory



and then it increases above the baseline case in the middle and later years, when her health is depreciating at its fastest rate. Figure 7 shows the implications of the difference in z/c ratios for the individual's lifetime trajectory of health capital. We see here that the effect of the change in the health investment trajectory is that the individual builds a lower stock of health capital than she does in the baseline case, a result that is not so surprising given the low value of the constant depreciation rate.

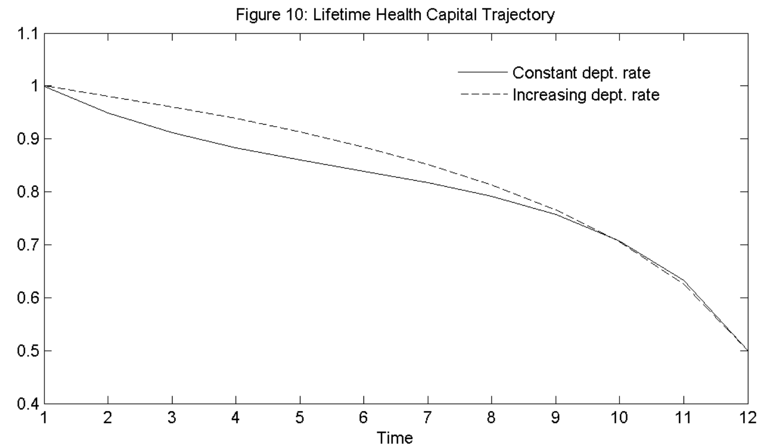
In the next set of simulation we set the value of the constant depreciation rate equal to the mean of the time varying depreciation series, as depicted in Figure 8. So given the values that we are using, an individual with age dependent depreciation rates will have lower depreciation rates than the constant case for more than half of lifetime and higher for the rest. Figure 9 reveals quite interesting dynamics for the z/c lifetime trajectory: when δ increases over time the individual invests more in health at the beginning and later periods of life and less in the middle periods compared to the case with a constant depreciation rate. Figure 10 depicts the outcomes for the individual's lifetime trajectory of health capital. As a result of the dynamics of the z/c ratio, the individual builds a higher stock health capital when depreciation rates vary with age and are lower for the first half of the life-cycle. Since we are using the same fixed endpoint for h in both cases the health stocks converge in later periods.



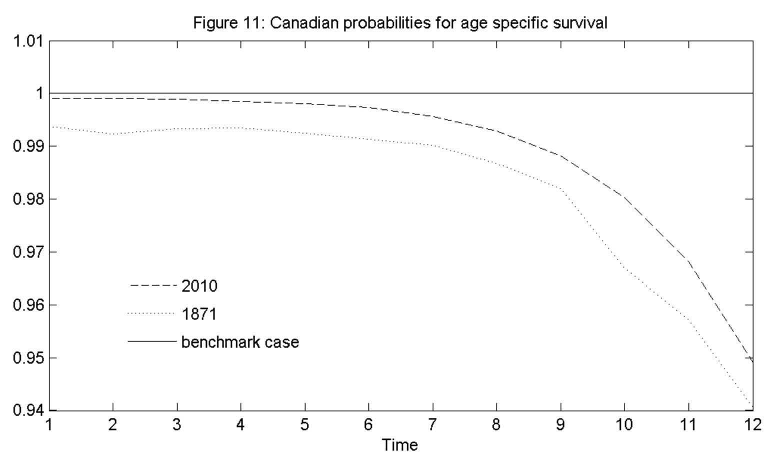


3.3. The case of non-constant survival probabilities

Next, we run simulations on the effect of introducing non-constant survival probabilities to the Grossman model. In the first sets of graphs below we establish a baseline by holding the rate of depreciation of health capital constant over time. Since this is an optimal control model we are modeling the plan formulated at the beginning of the planning horizon by



a forward looking individual. By the nature of our simulation exercise, we will assume that our individual will wind up living through the entire planning horizon, but the fact that she does not know that with certainty means that she will discount the future in a manner which reflects not just her pure rate of time preference but also her calculations of her survival probabilities. In a health investment model we cannot augment the pure rate of discount, since we have been assuming that she adopted exponential discounting, whereas a realistic schedule of age-specific survival probabilities (viewed from birth) will not be exponential: Canadian probabilities for age-specific survival are shown in Figure 11 below for two time periods, 1871 and 2010.



The non-exponential nature of the survival discount factor raises the likelihood of what would appear to be time-inconsistent behaviour even though the exponential nature of our individual's pure time discounting process will yield time consistent behaviour (see Strotz

(1956)). We intend to explore the implications of this fact in future simulation exercises.

In Figures 12 and 13 below we compare the trajectories for the z/c ratio and health capital for two cases, one in which survival is certain and the other in which we have assumed that the individual's survival probabilities were those of Canada in 1871.

Figure 12: Lifetime z/c Ratio Trajectory

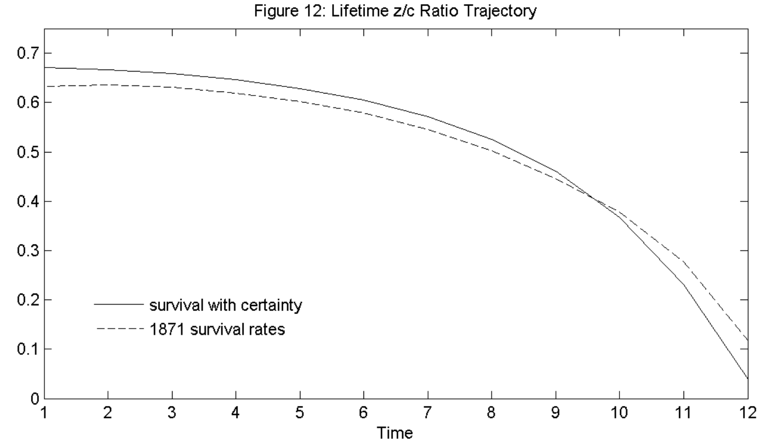
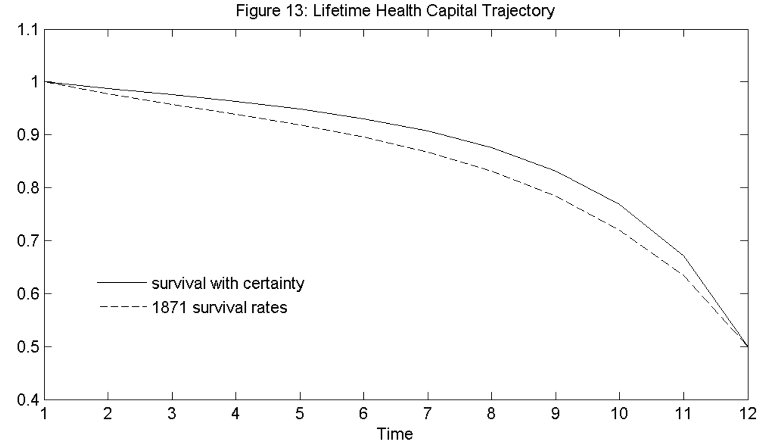


Figure 13: Lifetime Health Capital Trajectory



In terms of the values in our simulations, adding more realistic survival probabilities makes relatively little difference. We see that the effect of the greater degree of discounting is to reduce health investment throughout the life, with a consequent reduction in lifetime health status. This effect is consistent with the view that improvements in those factors which affect life expectancy but which are beyond the control of an individual, hence exogenous to our optimizing individual, will tend to encourage her to increase her own investment in health. The model which we are using here does not allow h to affect our individual's survival probabilities-modifying that would seem likely to produce a small reinforcing effect.[10]

Finally, in Figures 14 and 15 we compare our baseline model with one in which we have combined age-specific increasing rates of health depreciation and decreasing age-specific survival rates. The baseline model has a constant depreciation rate equal to the mean of the age increasing depreciation series, depicted in Figure 8, and survival rates are equal to 1.

Figure 14: Lifetime z/c Ratio Trajectory

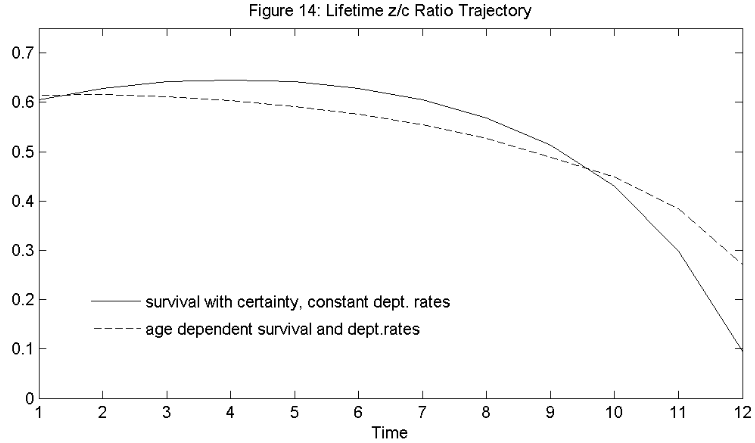
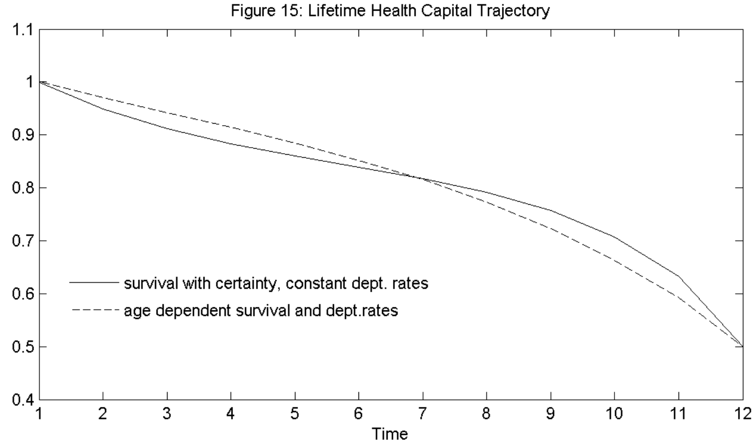


Figure 15: Lifetime Health Capital Trajectory



In the case with varying depreciation and survival rates the z/c ratio is higher in the first period and then below the case depicting constant rates, with the exception of the last few periods. Not surprisingly, given our results to this point, the results look very much like

[10]In future simulations we intend to explore the effect of adding realistic survival expectations interacting with other patterns of values for the remaining parameters.

those of the cases in which we have added only increasing depreciation rates. One difference is that when depreciation rate are constant and survival is certain, health capital is higher at the end of the lifetime.

4. Conclusion

Our argument in this paper has been two-fold: first that much of the richness of the Grossman model is in the intrinsic dynamics of the optimizing individual's optimal lifetime trajectories, and second that theoretical simulation is a useful device for focusing attention on the shape of those trajectories.[11] Theoretical simulation is widely used in other areas of theoretical economics, notably dynamic macroeconomics, and advances in the available software and programming techniques have greatly enhanced its potential for use in theoretical health economics. While it is true that simulation results are dependent on the parameter values assumed, sensitivity analysis can be used to evaluate the degree of that dependence, and the use of simulation can draw attention to the importance of certain assumptions-in our case the importance of the assumption of a fixed endpoint value of h for the optimal trajectories.[12] The ability to more fully characterize the dynamics of the individual's optimal plans suggests that theoretical simulation can inform econometric analysis by allowing the investigation of a whole range of factors within the Grossman model, in terms of their expected effect on individual health-related behaviours and ultimately the individual's lifetime health.

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[11] Especially important when we are considering empirical implementation of the model.

[12] See Forster (2001) for related analysis.

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Appendix

In this appendix, we provide a more detailed account of the solution algorithm used to solve the dynamic problem. The individual chooses $\{c_t\}_{t=1}^T$, $\{x_t\}_{t=1}^T$, $\{z_t\}_{t=1}^T$ and $\{h_{t+1}\}_{t=1}^{T-2}$ to maximize her expected discounted lifetime utility function. Her problem can be summarized as follows:

$$max_{c_t,x_t,z_t,h_{t+1}} E_1 \left\{ \sum_{t=1}^{T} \beta^{t-1} \Pi \left(\phi_s\right)_{s=1}^{t-1} U\left(x_t, c_t, h_t\right) \right\} \tag{9}$$

subject to the following constraints:

$$h_{t+1} = g\left(z_t\right) - f\left(c_t\right) + \left(1-\delta_t\right) h_t \tag{10}$$

$$y = p_x x_t + p_z z_t + p_c c_t \tag{11}$$

$$h_1 = \overline{h},\ h_T = hmin \tag{12}$$

$$c_t \geq 0,\ x_t \geq 0,\ h_t \geq 0 \tag{13}$$

Letting $X(c_t, z_t) = (y - p_z z_t - p_c c_t)\, p_x^{-1}$, the Lagrangean function takes the form:

$$\mathcal{L} = \sum_{t=1}^{T} \beta^{t-1} \Pi \left(\phi_s\right)_{s=1}^{t-1} \left\{ U\left(X(c_t, z_t), c_t, h_t\right) - \lambda_{t+1} \left[h_{t+1} - g\left(z_t\right) + f\left(c_t\right) - \left(1 - \delta_t\right) h_t\right] \right\}$$

Combining first order conditions we get:

$$U_x X_z + \left(\left(U_x X_c + U_x\right) f_c^{-1}\right) g_z = 0 \tag{14}$$

$$\beta \phi_t \left(U_h^+ - \left(1 - \delta_{t+1}\right) U_x^+ X_z^+ \left(g_z^+\right)^{-1} \right) + U_x X_z g_z^{-1} = 0 \tag{15}$$

where the + sign indicates functions updated for one period. Since finding the distributions of c_t, x_t, z_t and h_t from time 1 to T requires solving a large system of non-linear equations, we employ the recursive structure of the individual problem. These profiles are estimated by following a direct computation method, described in detail in Heer and Maussner (2005), which involve forward iterations on the first-order conditions that we obtain from the maximization problem (non-linear equations are solved using routines developed by Kelley (2003) and Miranda and Fackler (2002)). The solution algorithm that we have followed is described in steps below:

Step 1. Guess the initial level of the health investment z_1: given endowments h_1 and y and initial iterate z_1, find c_1 with the help of equation 14. Find x_1 using the period budget constraint, equation 11.

Step 2. Given values for the first period, we can calculate the level of health capital in the next period h_2 using equation 10.

Step 3. Given values for c_1, z_1, x_1 and h_2, the system of the two non-linear equations 14 and 15 is used to solve for the two unknowns z_2 and c_2. Similarly, x_2 is computed by using the period budget constraint.

Step 4. Repeat step 2 to compute h_{t+1} and step 3 to compute z_{t+1} and c_{t+1}, given values for c_t, z_t, x_t and h_t- continue this way until period T.

Step 5. Check whether $h_T - hmin < \epsilon$, where ϵ denotes the level of tolerance (a value of 0.00001 in our estimation). If this condition is met than the problem is solved, otherwise update the initial iterate in step 1 and repeat until convergence.